New dispersion relations in description of $\pi\pi$ scattering amplitudes

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- Motivation
- Roy's equations
  - -few words on derivation
  - -example of analysys using Roy's equations
- One substracted dispersion relations (1S)
- Evaluation and comparison of errors in Roy's and 1S equations
- Further works on the errors
- Conclusions

MOTIVATIONS

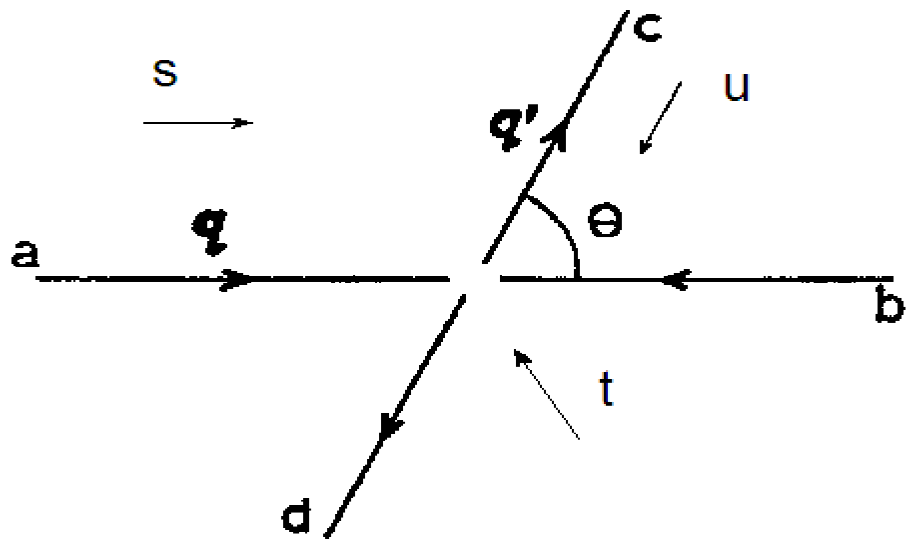
Why pions?

- Pions are the lightest mesons so very often appear in reactions and decays.
- Threshold parameters are closely related with e.g. quark condensate
- Knowledge of the pion-pion interaction up to about 2 GeV let us study the spectrum of heavier mesons (scalar ones for example).

MOTIVATIONS

Roy's observation and paper in 1971:
"*Exact integral equation for pion pion scattering involving only physical region partial waves„*

amplitude in s-channel should be the same in the t and u-channell (crossing symmetry)



MOTIVATIONS

Problem of the errors of pion-pion amplitudes:

- Swiss group: J. Gasser, H. Leutwyler,
  B. Ananthanarayan, G. Colangelo.
  First their paper on model with Roy's equations:
  "*Roy equation analysis of pi pi scattering*" '2001
- 15 other papers - at least 5 of them are the answers
  for questions in papers of the Spanish group:
  F. Yndurain, J. R. Pelaez, R. Garcia-Martin and
  R. Kaminski (10 papers on dispersion relations in
  description of the pion-pion scattering amplitudes).

DISPERSION RELATIONS

$$\int_C dz' \frac{f(z')}{z'-z} = 2\pi i f(z) \qquad (1)$$

$$\mathrm{Re}\, f(z) = \frac{1}{2\pi} \int_C dz' \frac{\mathrm{Im}\, f(z')}{z'-z} \qquad (2)$$

- One subtraction:

$$\mathrm{Re}\, T(z) - \mathrm{Re}\, T(z_1) = \frac{z_1 - z}{2\pi} \int_C dz' \frac{\mathrm{Im}\, T(z')}{(z'-z)(z'-z_1)} \qquad (3)$$

- Two subtractions:

$$\mathrm{Re}\, T(z) = g(z_1, z_2) + h(z_1, z_2) z \qquad (4)$$

$$+ \frac{(z_1 - z)(z_2 - z)}{2\pi} \int_C dz' \frac{\mathrm{Im}\, T(z')}{(z'-z)(z'-z_1)(z'-z_2)}$$

ROY'S EQUATIONS CROSSING SYMETRY

$$\mathrm{Re}\,T(s,t,u)=\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,T(s',t,u')}{s'^2}\left(\frac{s^2}{s'-s}1^{II'}+\frac{u^2}{s'-u}C^{II'}{}_{SU}\right)+C(t) \quad (5)$$

$$T(s=0,t=t_0,u=4m_\pi^2-t_0)=T(s=t_0,t=0,u=4m_\pi^2-t_0) \quad (6)$$

so:

$$\mathrm{Re}\,T(0,t,4m_\pi^2-t)=\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,T(s',t)}{s'^2}\frac{(4m_\pi^2-t)}{s'-4m_\pi^2-t}+C(t) \quad (7)$$

and:

$$\mathrm{Re}\,T(t,0,4m_\pi^2-t)=\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,T(s',0)}{s'^2}\left(\frac{t^2}{s'-t}\frac{(4m_\pi^2-t)}{s'-4m_\pi^2-t}\right)+C(0) \quad (8)$$

ROY'S EQUATIONS CROSSING SYMETRY

$$T(4m_\pi^2,0,0) = a_0 = \frac{1}{3}\left(a_0^0 + 2a_0^2\right) \qquad (9)$$

so:

$$a_0 = C(0) + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\operatorname{Im} T(s',0)}{s'^2}\frac{16m_\pi^4}{s' - 4m_\pi^2} \qquad (10)$$

After projection on partial waves

$$f_L^I = \int_0^1 d\cos(\theta) P_L\left(\cos(\theta)\right) T^I(s,t) \qquad (11)$$

ROY'S EQUATIONS

and after isospin decomposition:

$$\mathrm{Re}\, f_L^I(s) = a_0^0\delta_{I0}\delta_{l0} + a_0^2\delta_{I2}\delta_{l0} \qquad (12)$$

$$+(a_0^0 - 5a_0^2)\left(\delta_{I0}\delta_{L0} + \frac{1}{6}\delta_{I1}\delta_{L1} - \frac{1}{2}\delta_{I2}\delta_{L0}\right) \times \frac{s - 4m_\pi^2}{12m_\pi^2} \leftarrow \text{ST}$$

Kernel Term→

$$+\sum_{I'=0}^{2}\sum_{L'=0}^{1}\int_{4m_\pi^2}^{s_{\max}} ds' K_{LL'}^{II'}(s,s')\,\mathrm{Im}\, f_L^I(s) + d_L^I(s, s_{\max}) \leftarrow \text{DT}$$

a_0^0, a_0^2 S0 and S2 scattering lenghts(in substracting terms)

$K_{LL'}^{II'}(s,s')$ kernels

$d_L^I(s, s_{\max})$ „driving terms"

-higher energy parts of the S0, P and S2-waves

-higher energy dependence of higher partial waves

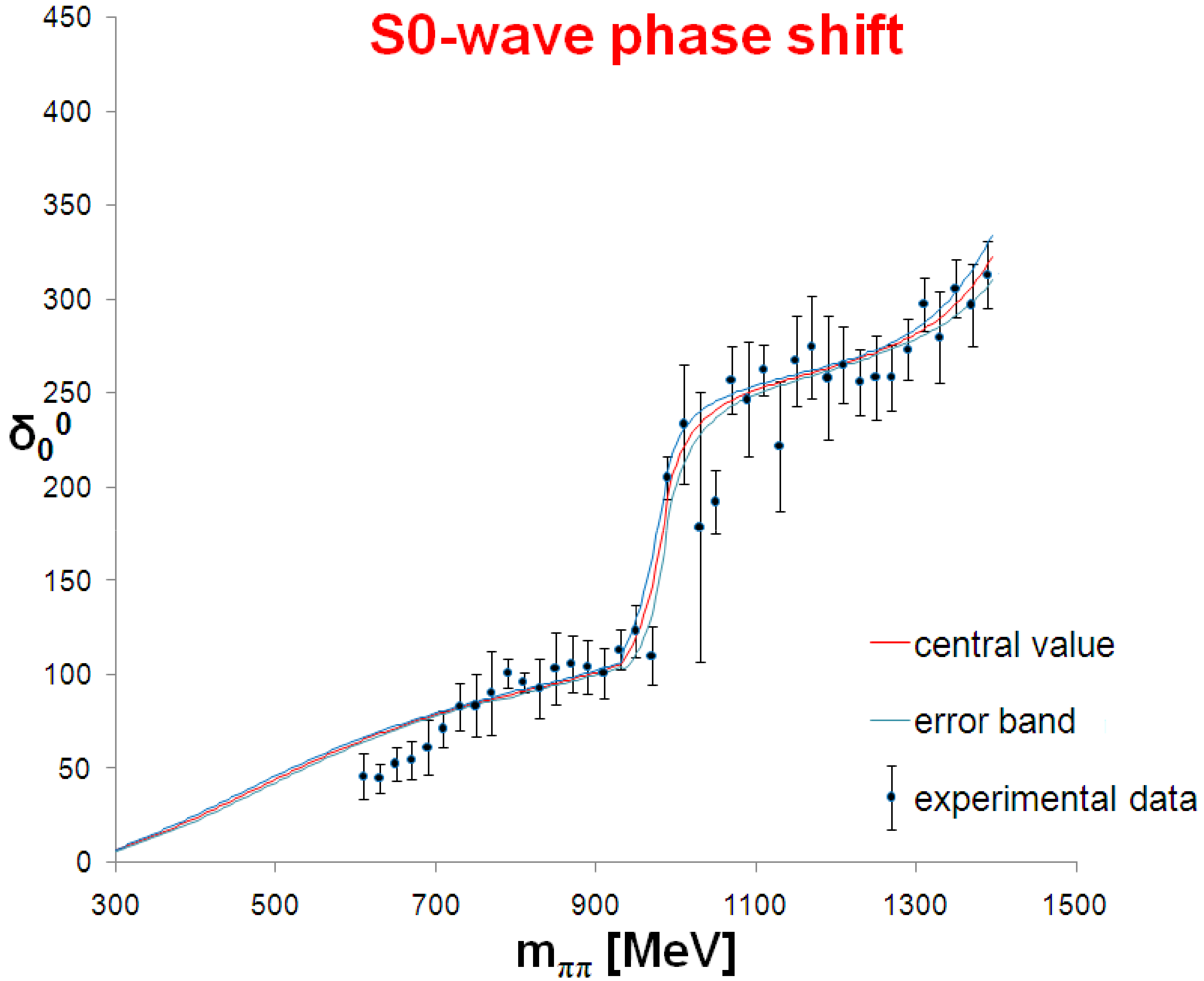
ROY'S EQUATIONS

„Input"

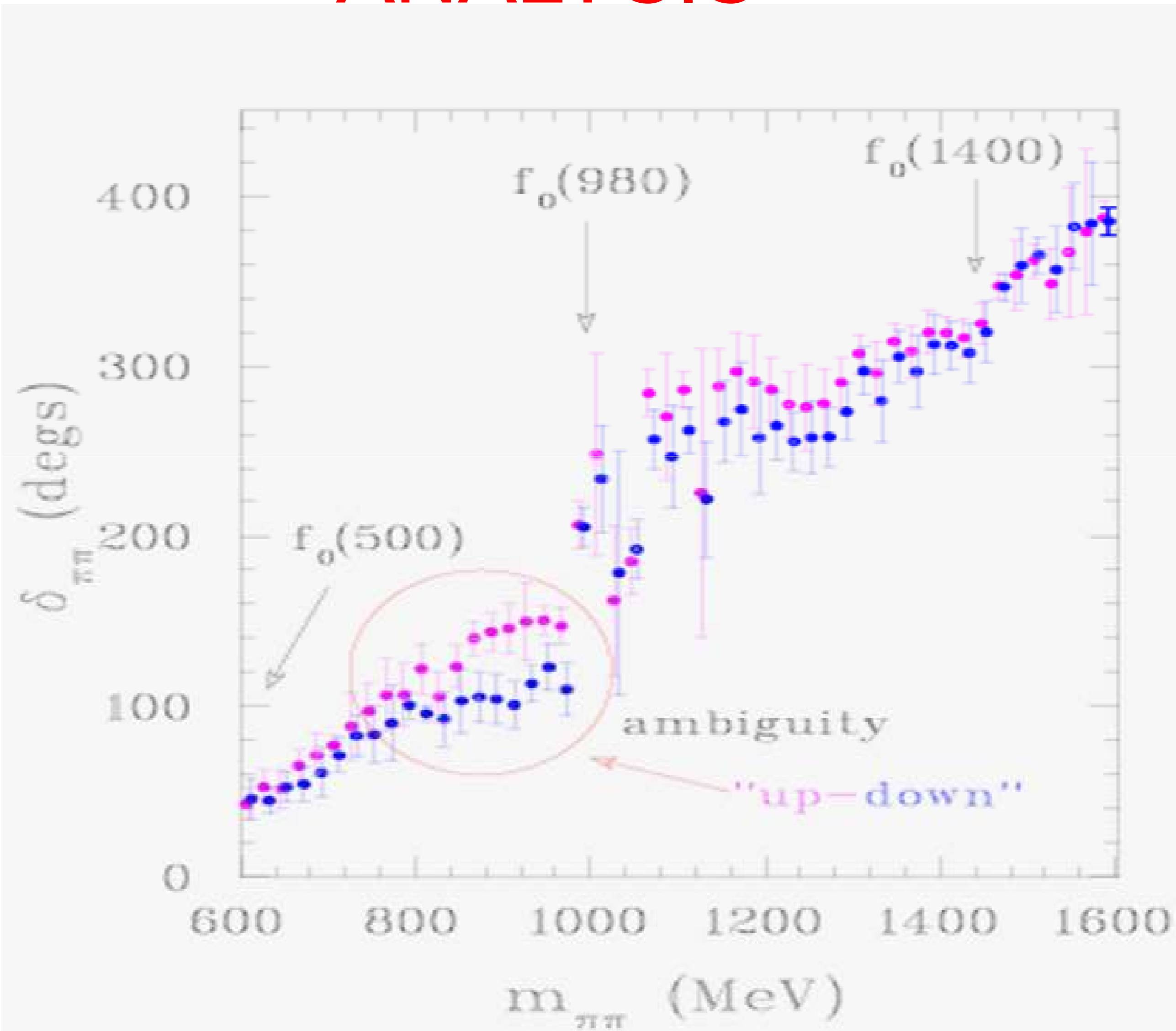
$$f_L^I = \sqrt{\frac{s}{s-4m_\pi^2}}\,\frac{1}{2i}\left(\eta_L^I e^{2i\delta_L^I} - 1\right) \qquad (13)$$

$$\left.\begin{array}{l}\eta_L^I:\ \text{inelasticities}\\ \delta_L^I:\ \text{phase shifts}\end{array}\right\}\text{From model predictions fitted to experimental data}$$

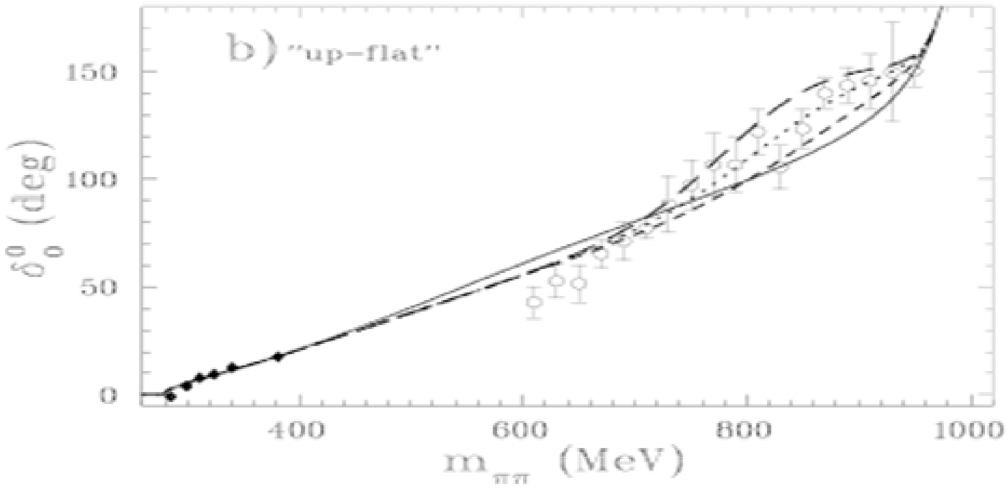
S0-wave phase shift

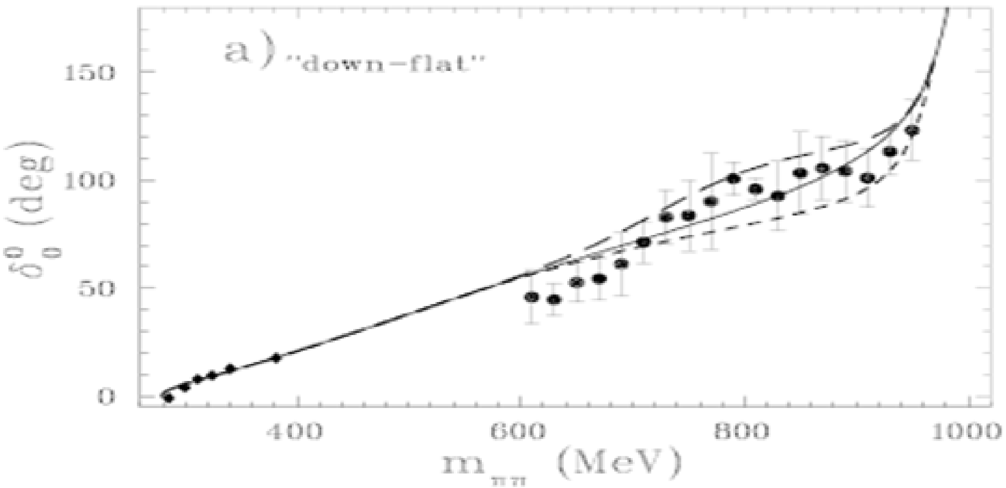


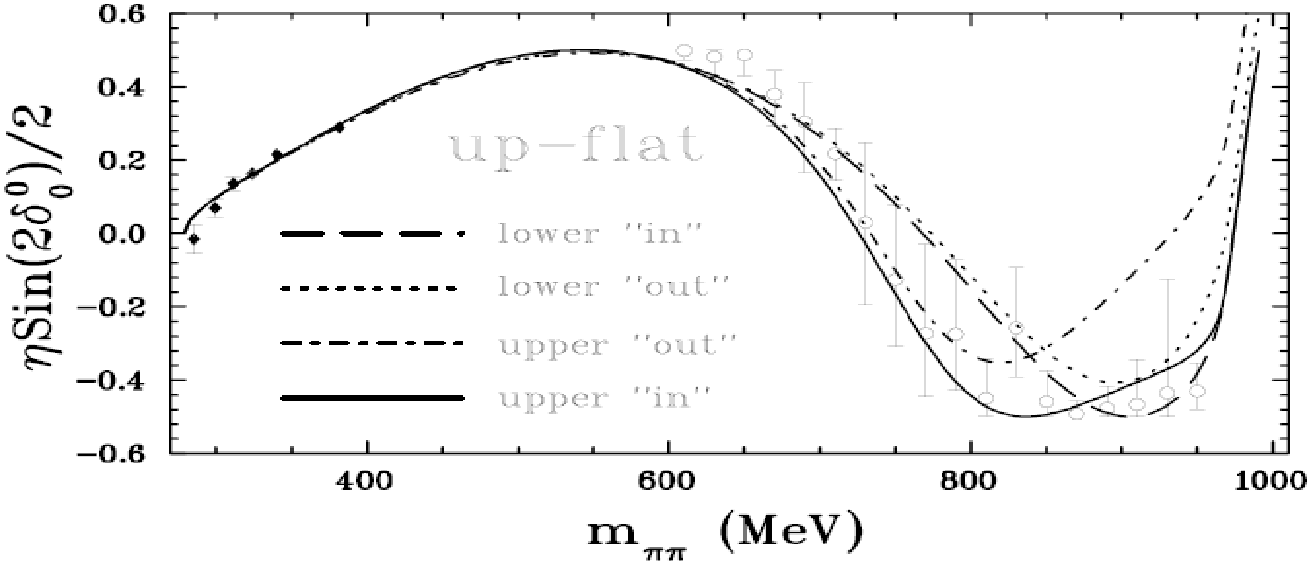
ROY'S EQUATION USED IN ANALYSIS

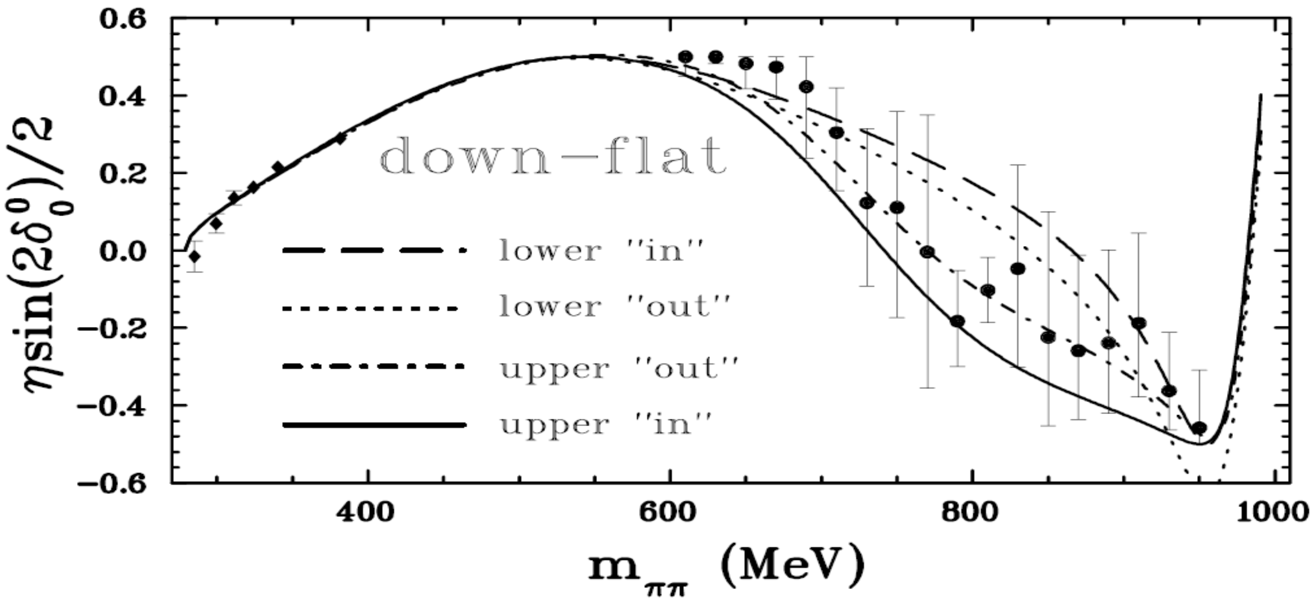


ROY'S EQUATION USED IN ANALYSIS






0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
400
600
800
1000
$\eta\text{Sin}(2\delta^0_0)/2$
$m_{\pi\pi}$ (MeV)
up−flat
lower "in"
lower "out"
upper "out"
upper "in"


0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
400
600
800
1000
$\eta\sin(2\delta^0_0)/2$
$m_{\pi\pi}$ (MeV)
down−flat
lower "in"
lower "out"
upper "out"
upper "in"

1S EQUATIONS

$$\mathrm{Re}\,T(z) - \mathrm{Re}\,T(z_1) = \frac{z_1 - z}{2\pi}\int_C dz' \frac{\mathrm{Im}\,T(z')}{(z'-z)(z'-z_1)} \qquad (14)$$

$$\mathrm{Re}\, f_L^I(s) = \frac{8}{\pi}\sum_{I''} C^{st}_{II''} a_0^{I''} \quad \leftarrow \text{ST} \qquad (15)$$

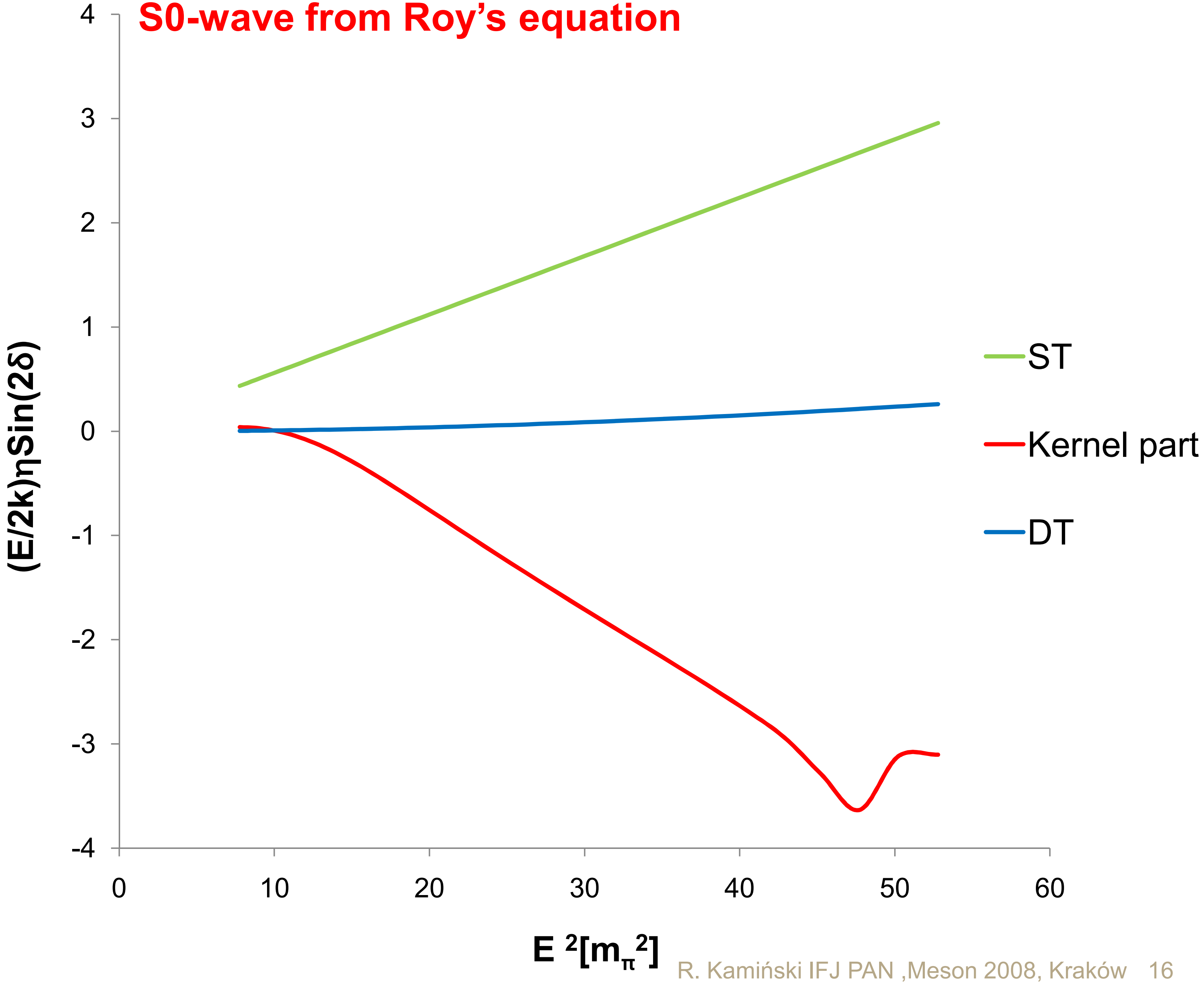
Kernel→ Term

$$+\sum_{L'}(2L'+1)\int_4^{sh} ds' \Big\{ K_{LL'}(s,s')\,\mathrm{Im}\, f_{l'}^I(s') - J_{LL'}(s,s')\sum_{I'} C^{su}_{II'} f_{L'}^I(s')$$

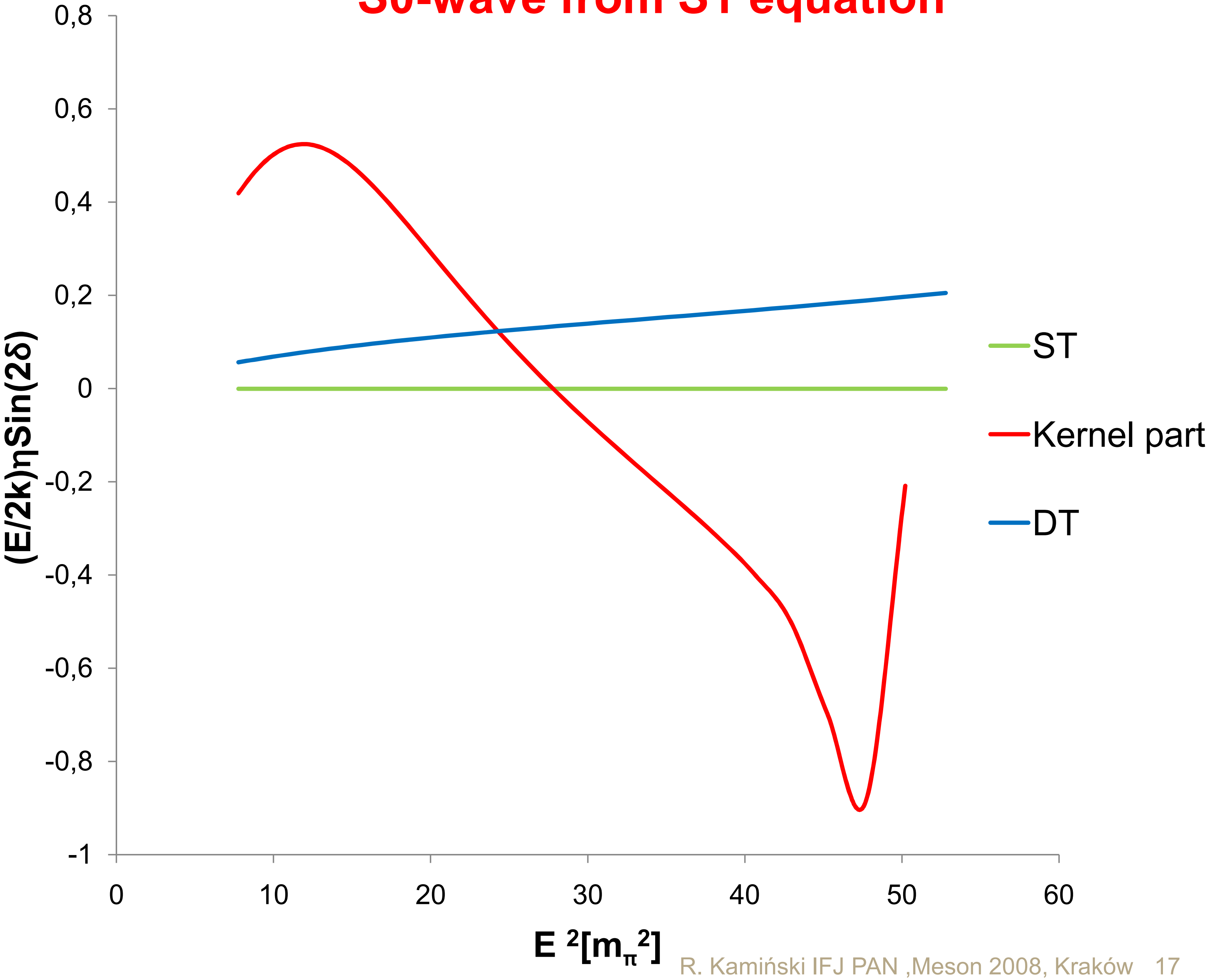
$$+\sum_{I''} C^{st}_{II''}\Big[M_{l(s,s')}\,\mathrm{Im}\, f_{L'}^{I''} - N_L(s,s')\sum_{I'''} C^{su}_{I''I'''} f_{L'}^{I'''}(s') \Big]\Big\}$$

$$+\mathrm{Re}\, f_L^{(h.e.),I}(\mathrm{s}) \quad \leftarrow \text{DT}$$

S0-wave from Roy's equation



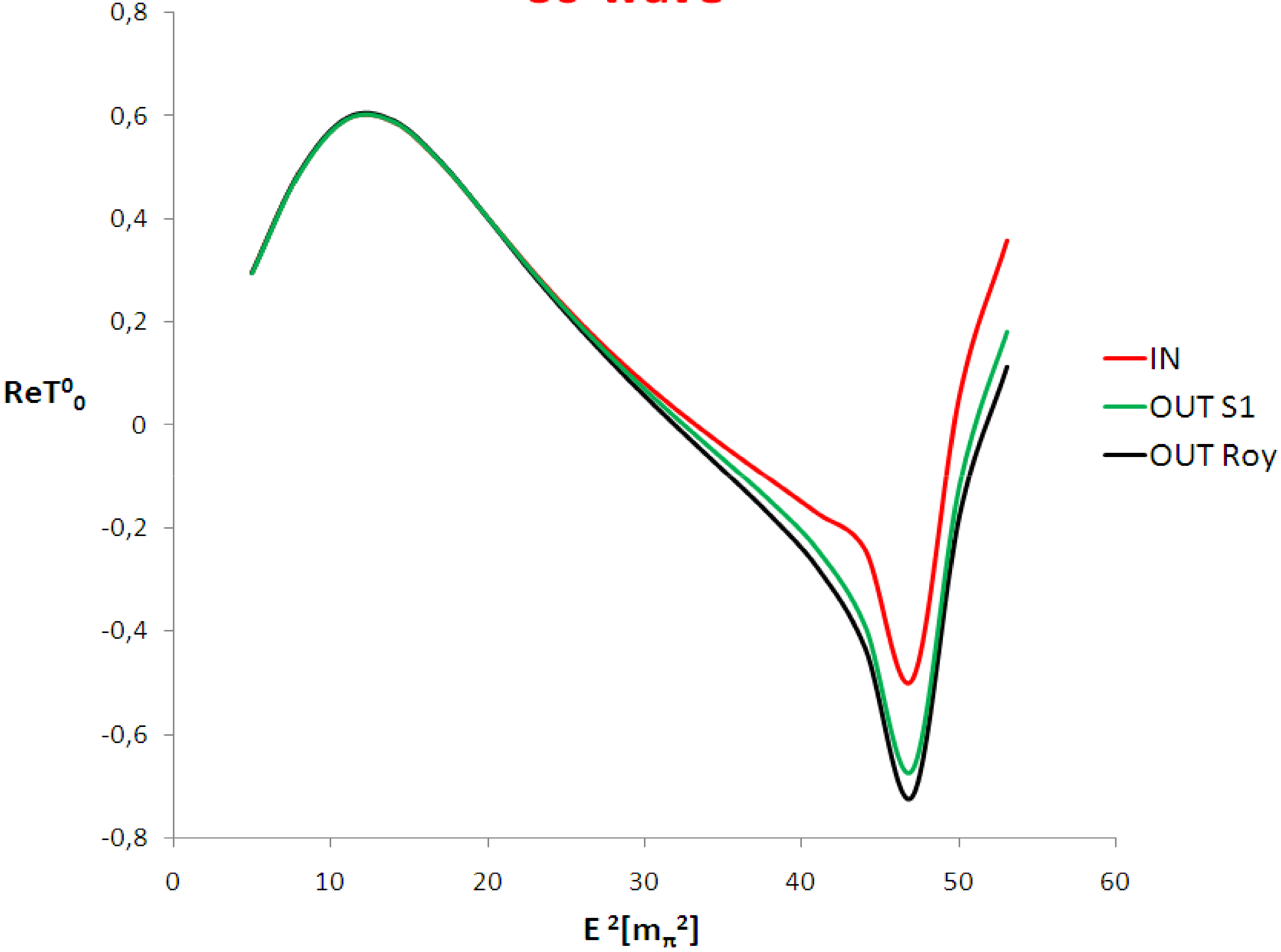
S0-wave from S1 equation



SUBTRACTING TERMS

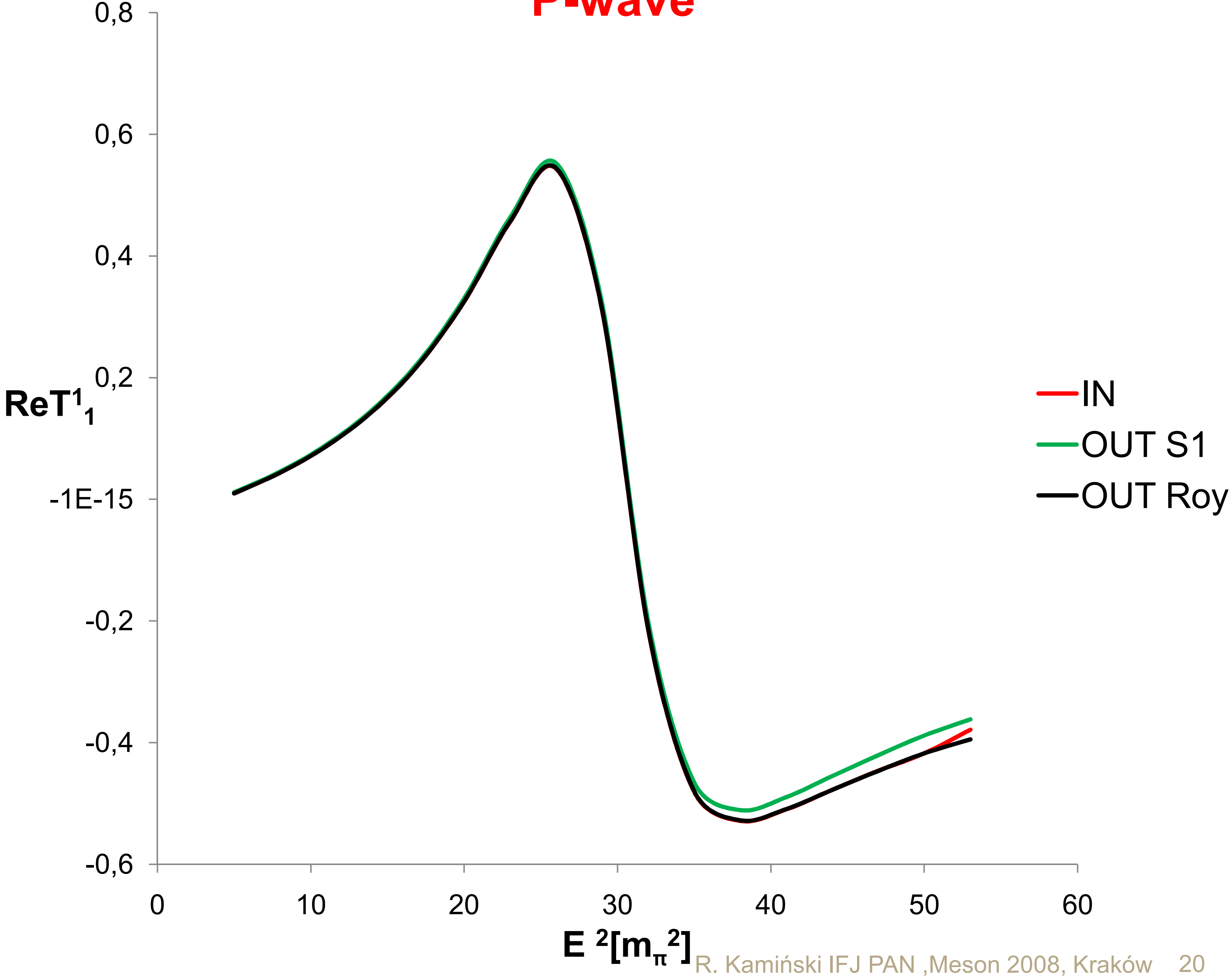
	Roy's equaton	1S equaton
S0-wave	$a_0^0 + (2a_0^0 - 5a_2^0)(s-4)/12$	$(a_0^0 + 5a_2^0)/3$
P1-wave	$(2a_0^0 - 5a_2^0)(s-4)/72$	$(a_0^0 - 5/2\,a_2^0)/3$
S2-wave	$a_0^0 + (2a_0^0 - 5a_2^0)(s-4)/24$	$(a_0^0 + 1/2\,a_2^0)/3$

S0-wave



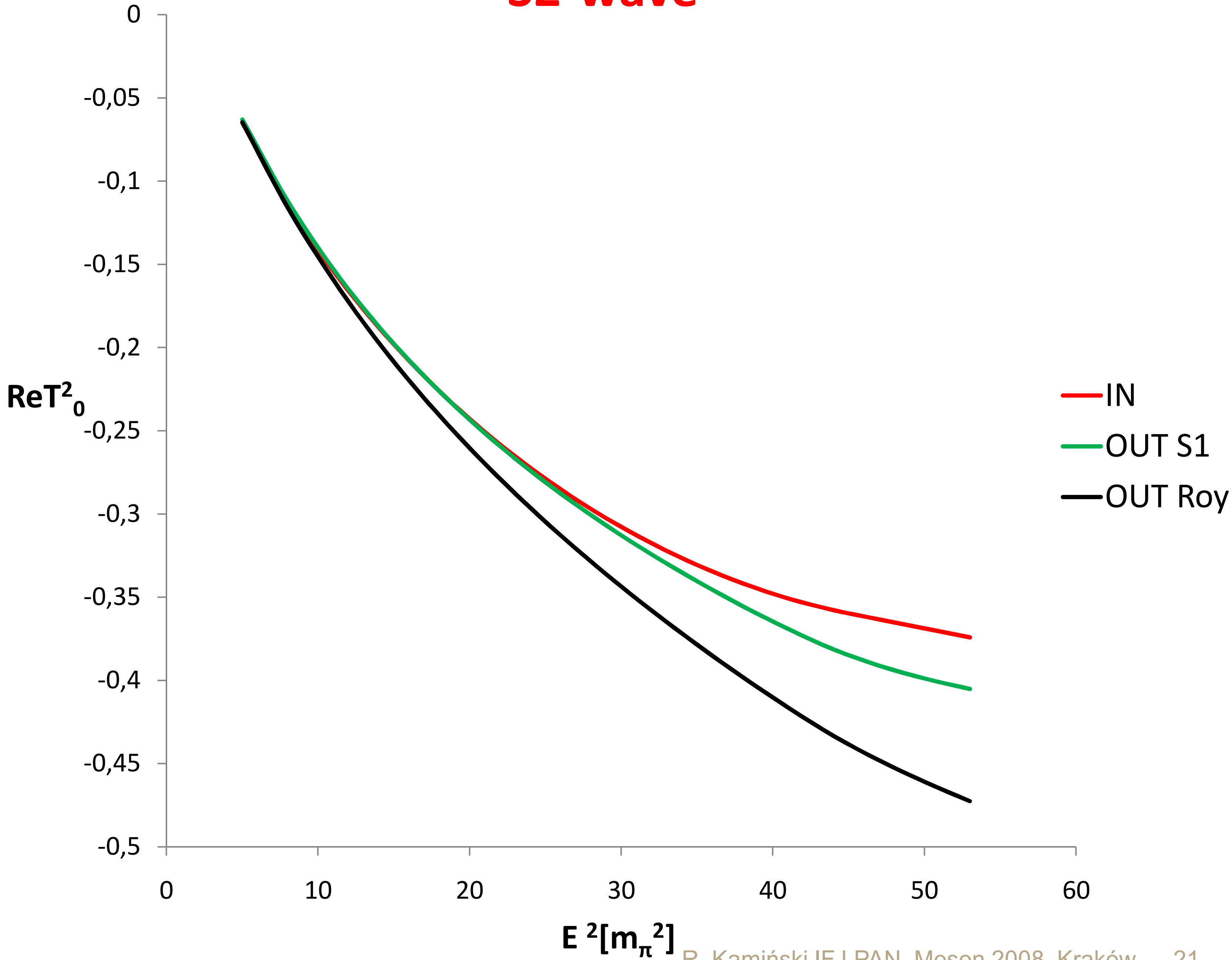
0,8
0,6
0,4
0,2
0
-0,2
-0,4
-0,6
-0,8
ReT$^0_0$
0
10
20
30
40
50
60
E ^{2}[m$_\pi$^{2}]
IN
OUT S1
OUT Roy

P-wave



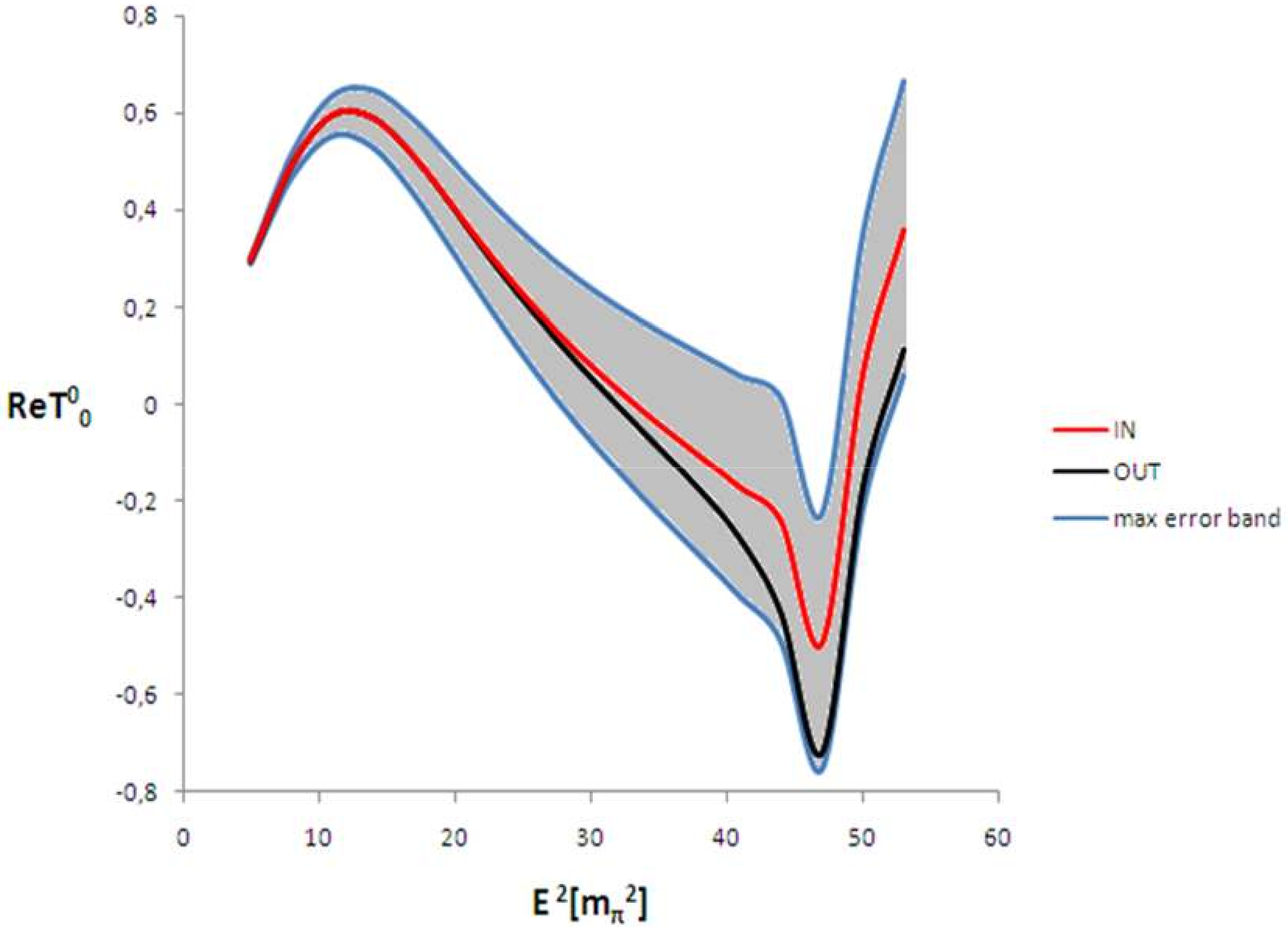
0,8
0,6
0,4
0,2
-1E-15
-0,2
-0,4
-0,6
ReT$^1_1$
0
10
20
30
40
50
60
E ^{2}[m$_\pi{}^2$]
IN
OUT S1
OUT Roy

S2-wave

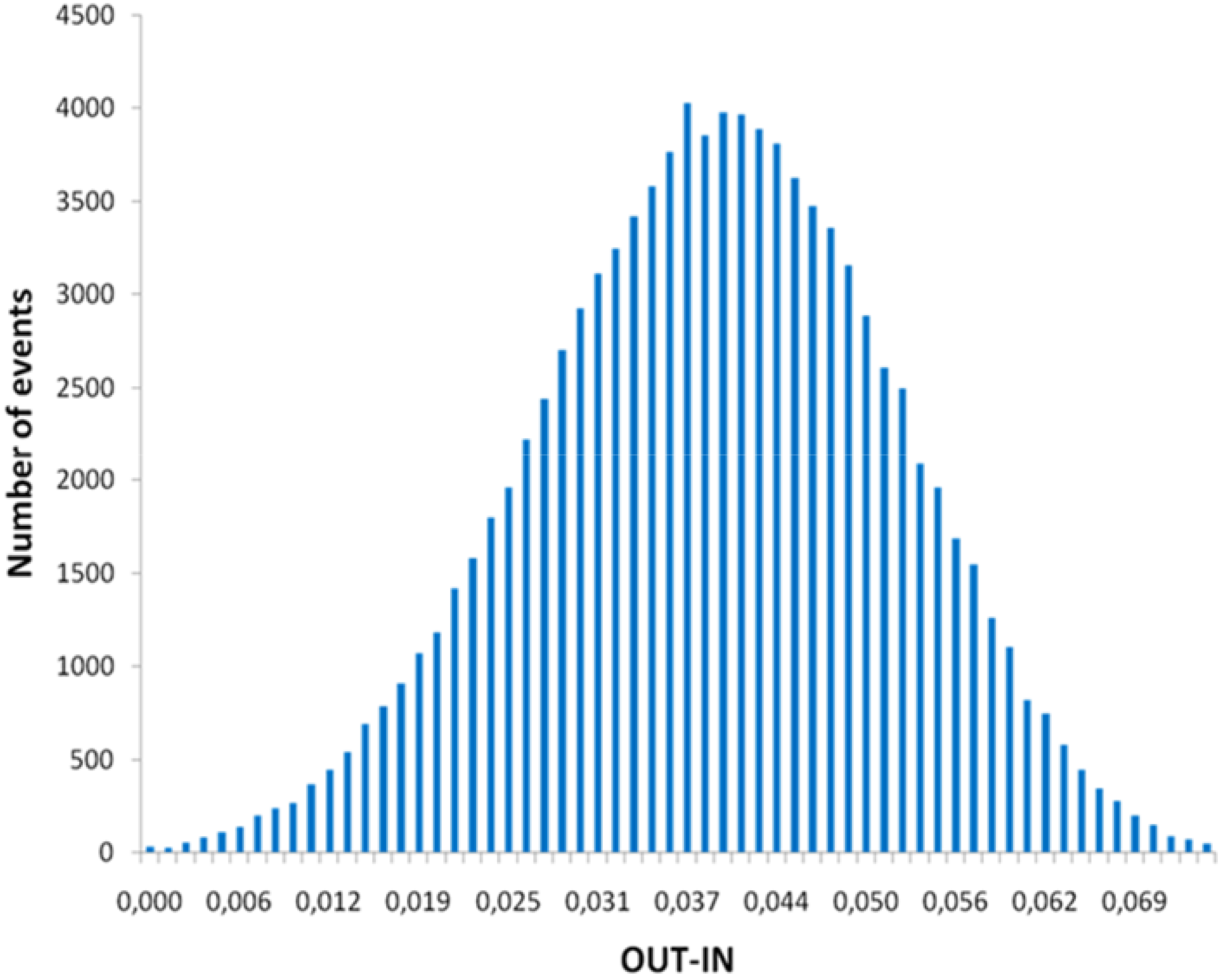


0
-0,05
-0,1
-0,15
-0,2
-0,25
-0,3
-0,35
-0,4
-0,45
-0,5
ReT²₀
0
10
20
30
40
50
60
E ²[mπ²]
IN
OUT S1
OUT Roy

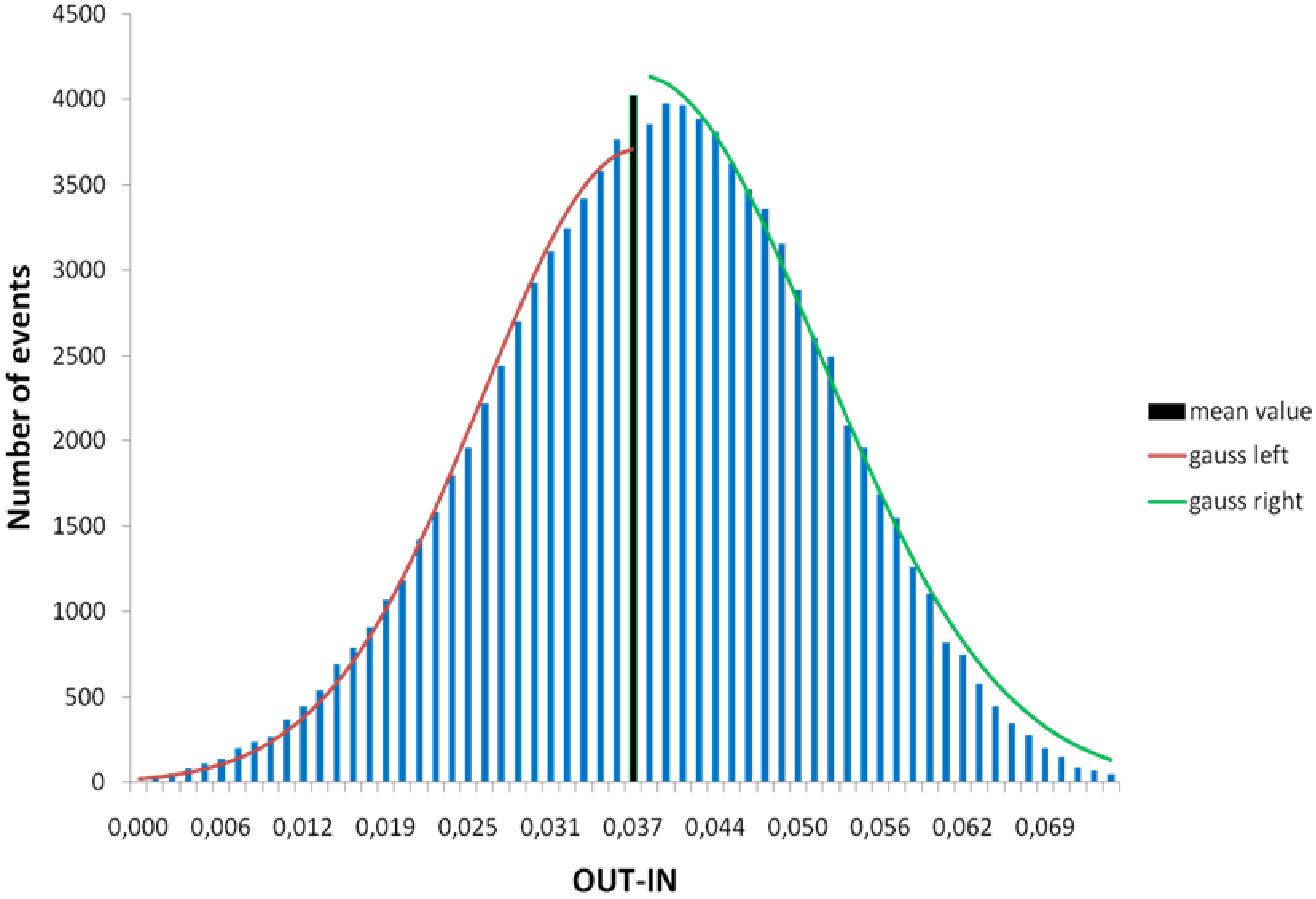
S0-wave from Roy's equation with maximal errors



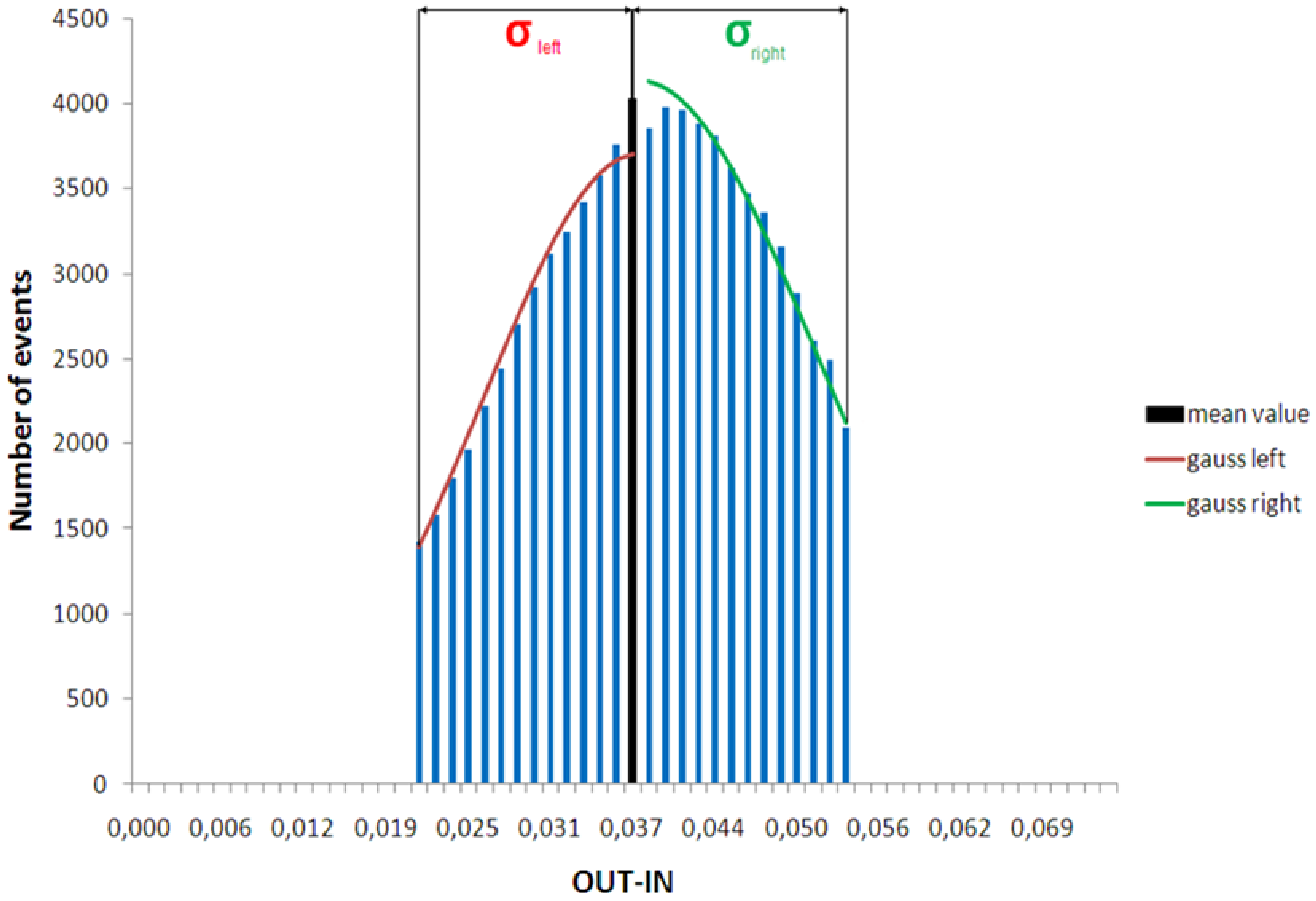
S0-wave from Roy 's equation
S=20



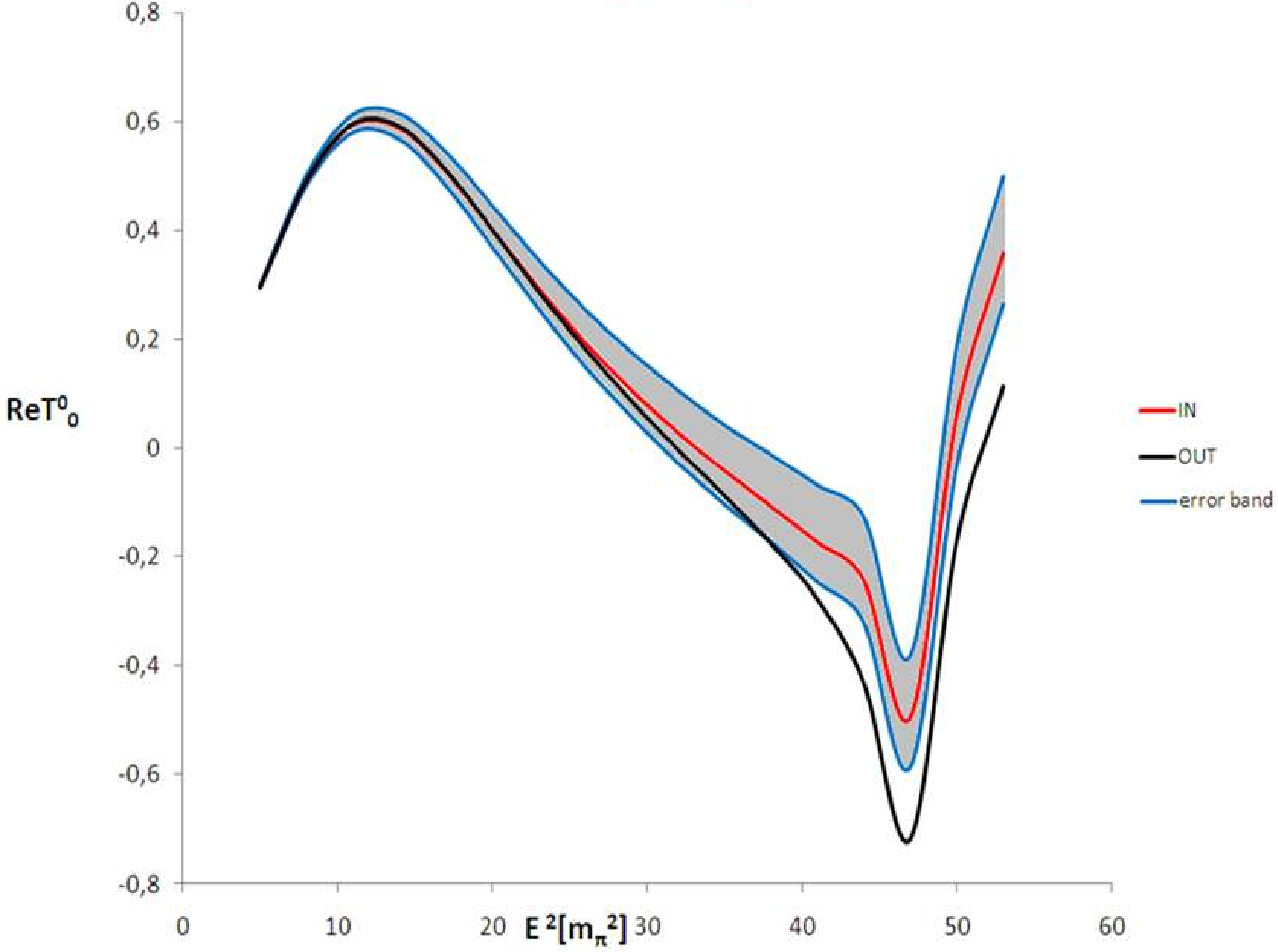
S0-wave from Roy 's equation
S=20



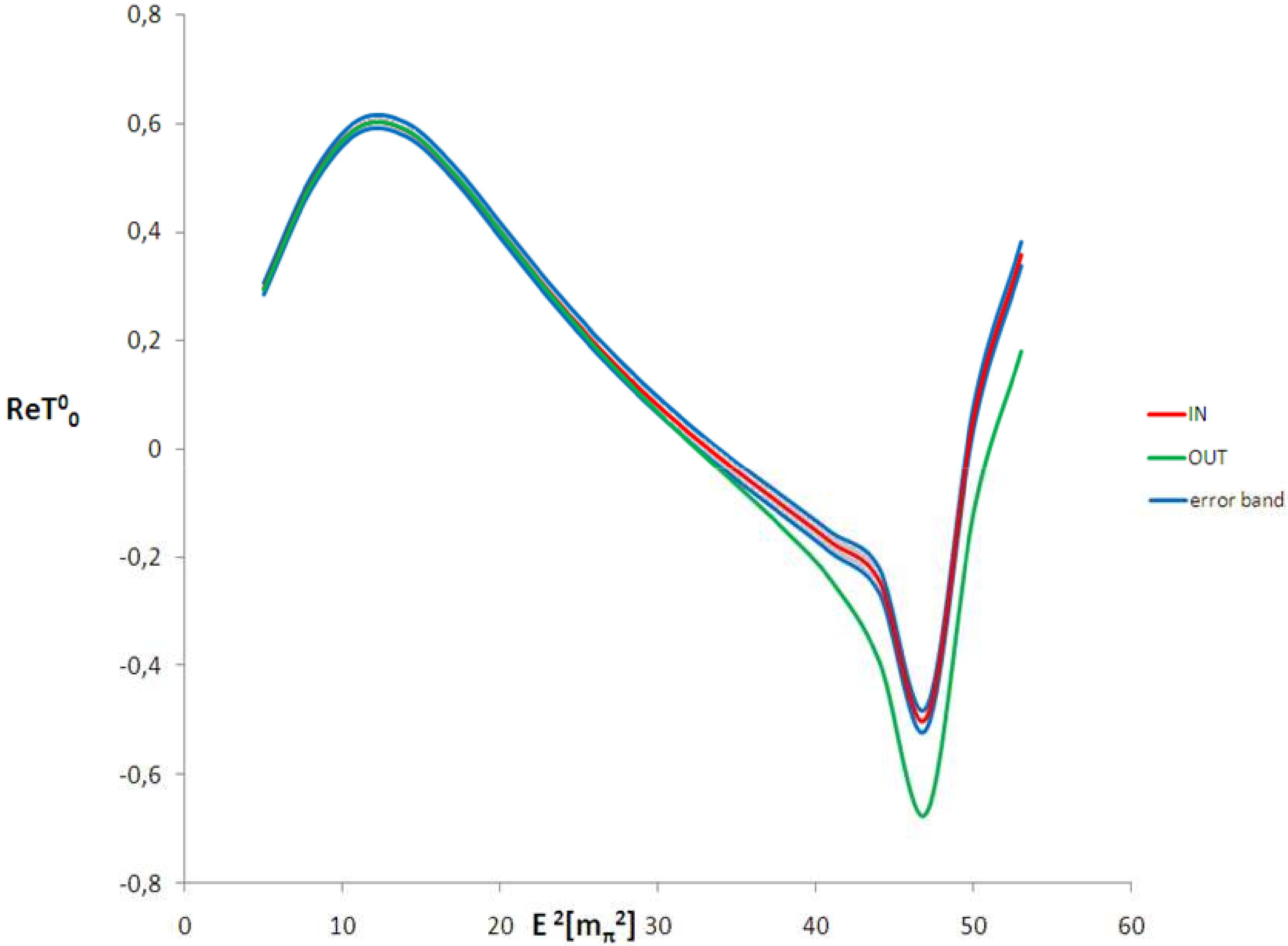
S0-wave from Roy 's equation
S=20



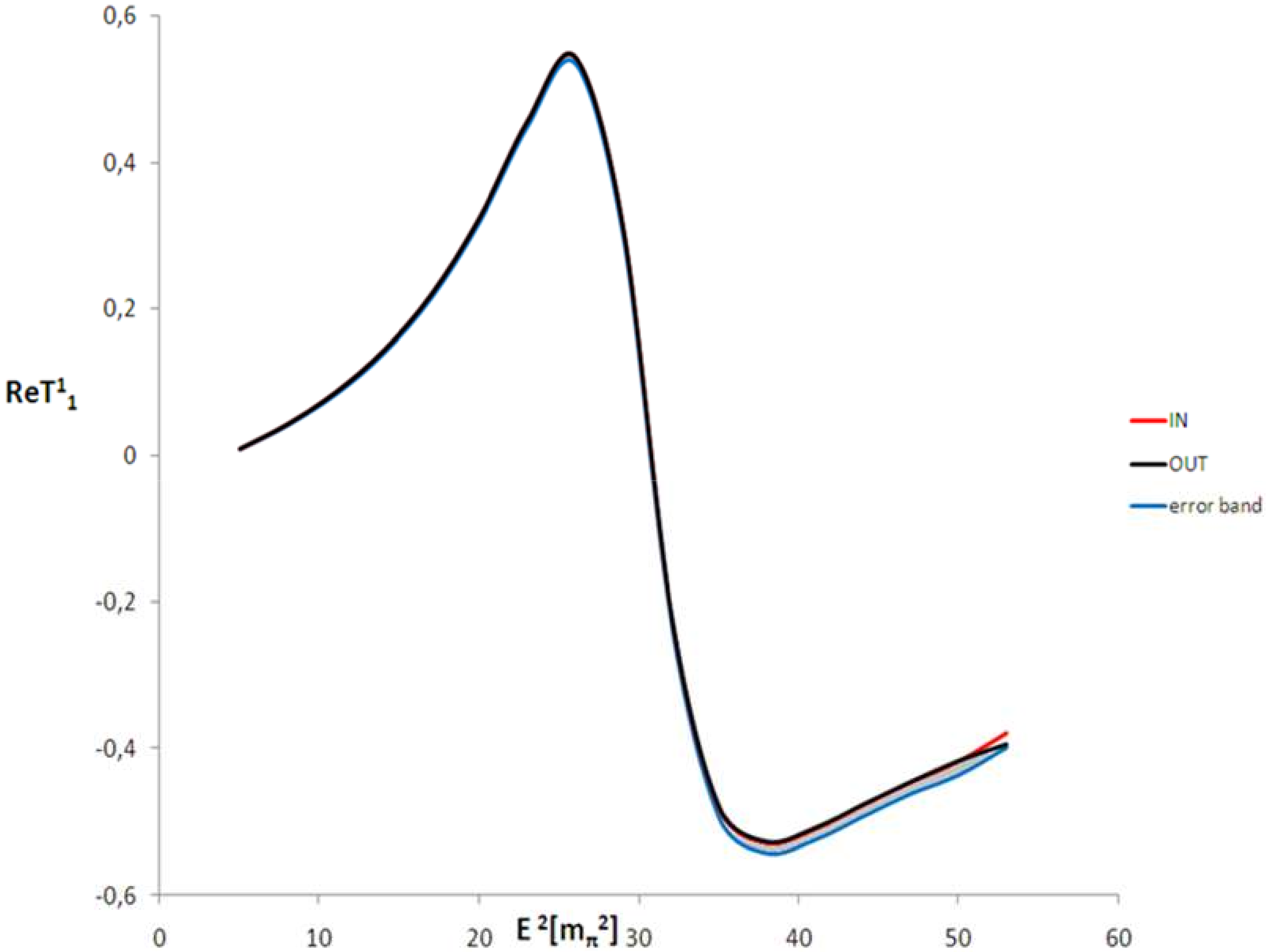
S0-wave from Roy's equation with errors



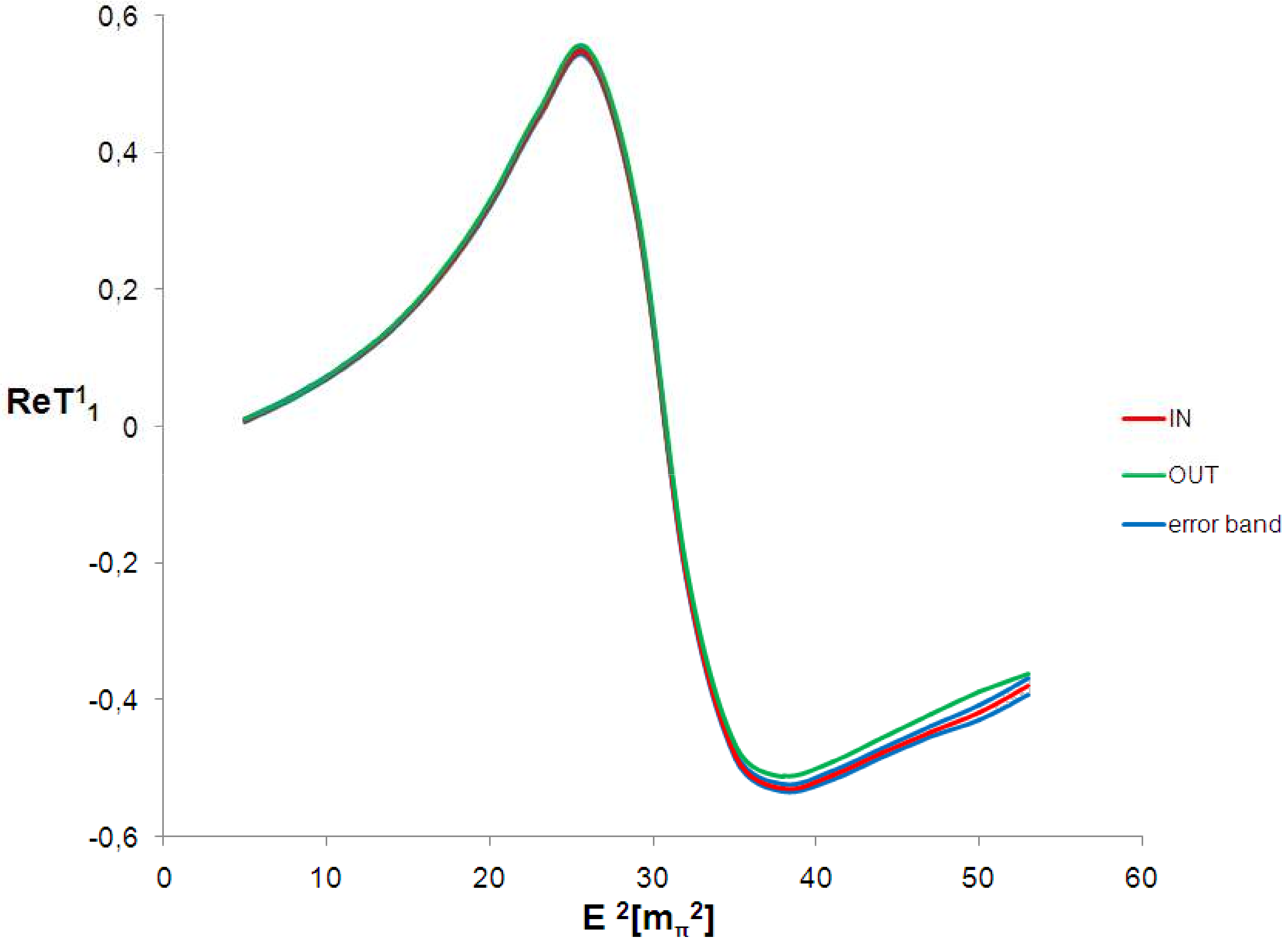
S0-wave from 1S equation with errors



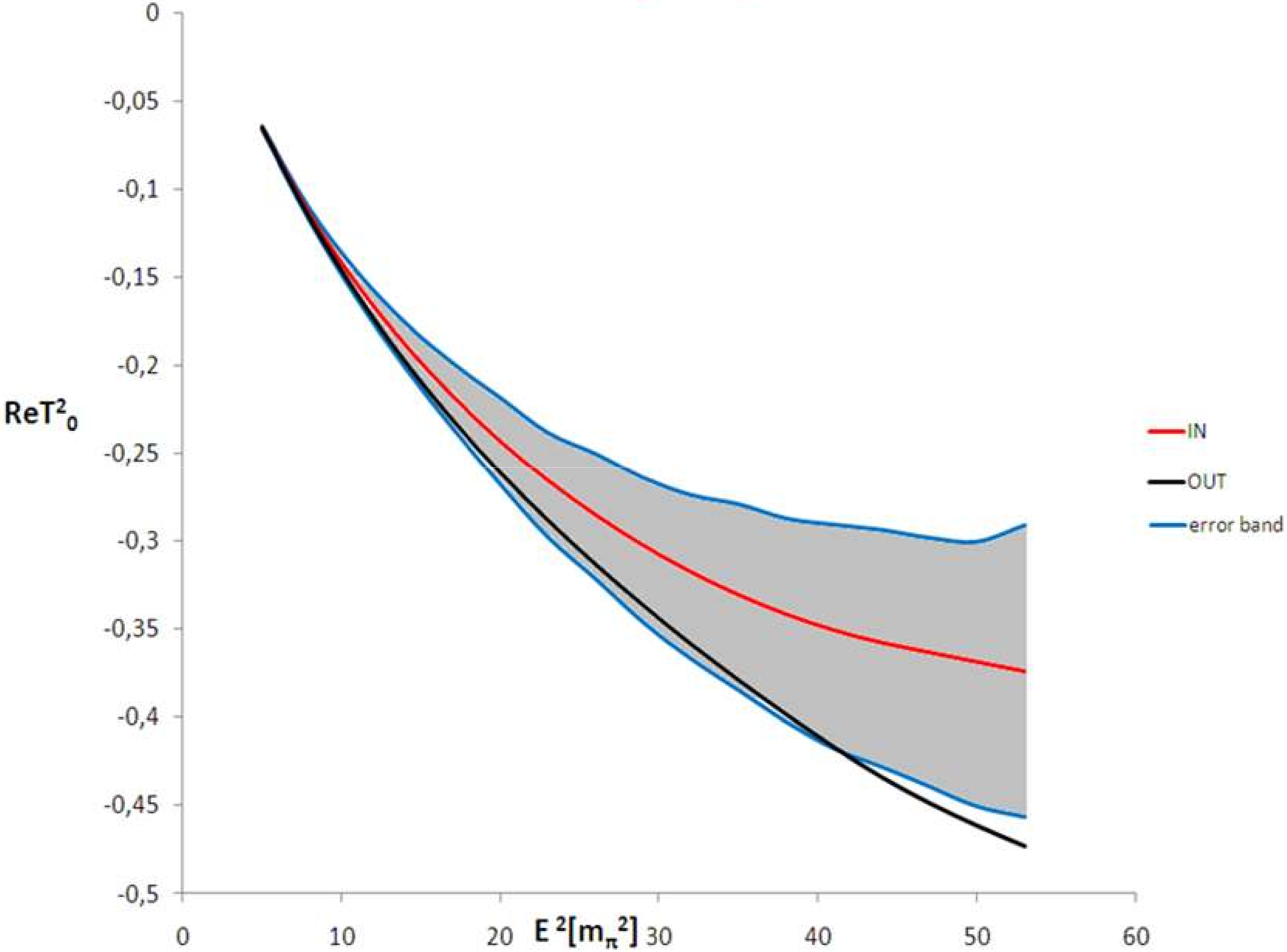
P-wave from Roy's equation with errors



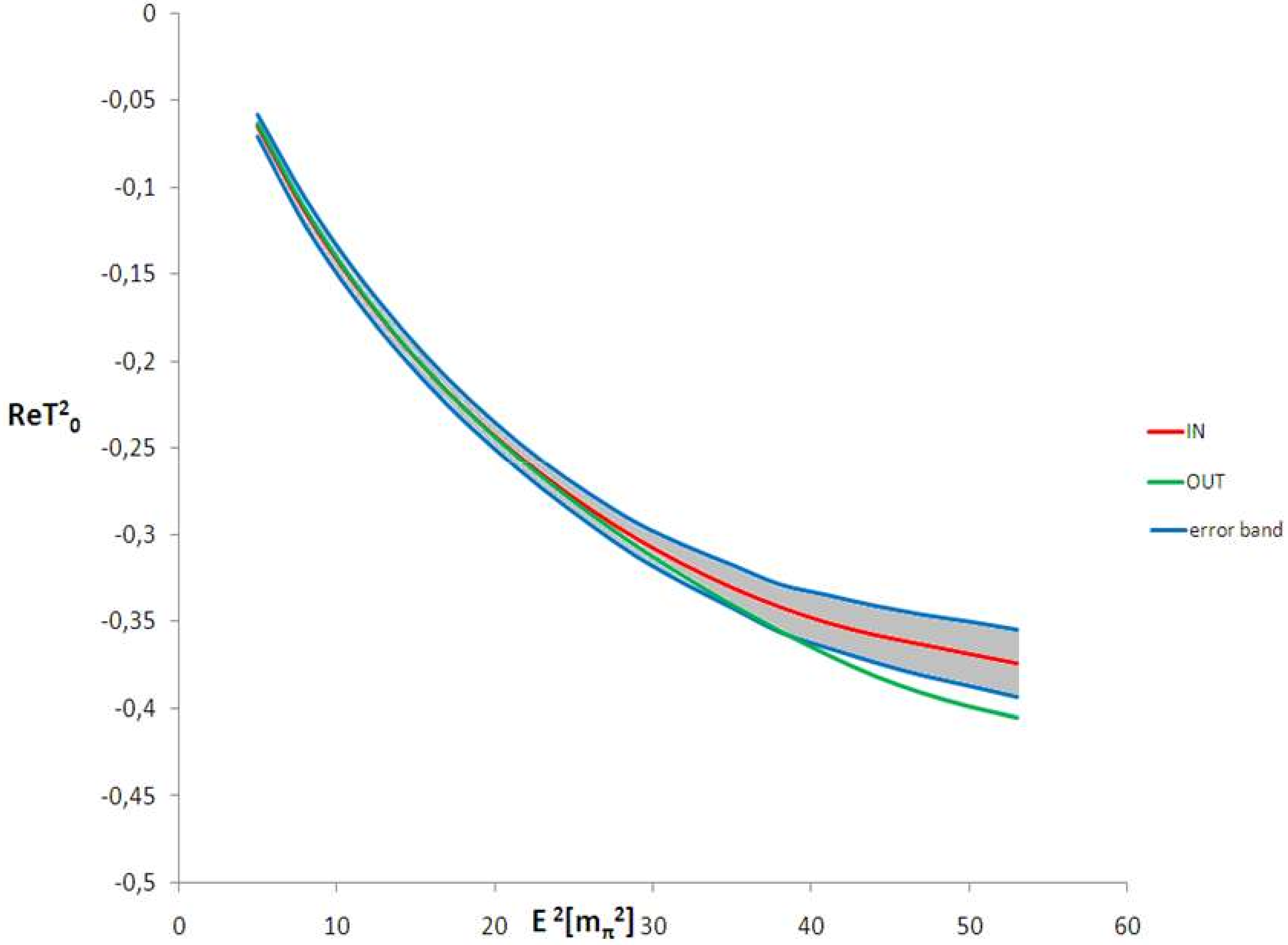
P-wave from S1 equation with errors



S2-wave from Roy's equation with errors



S2-wave from 1S equation with errors



ERROR COMPARISON

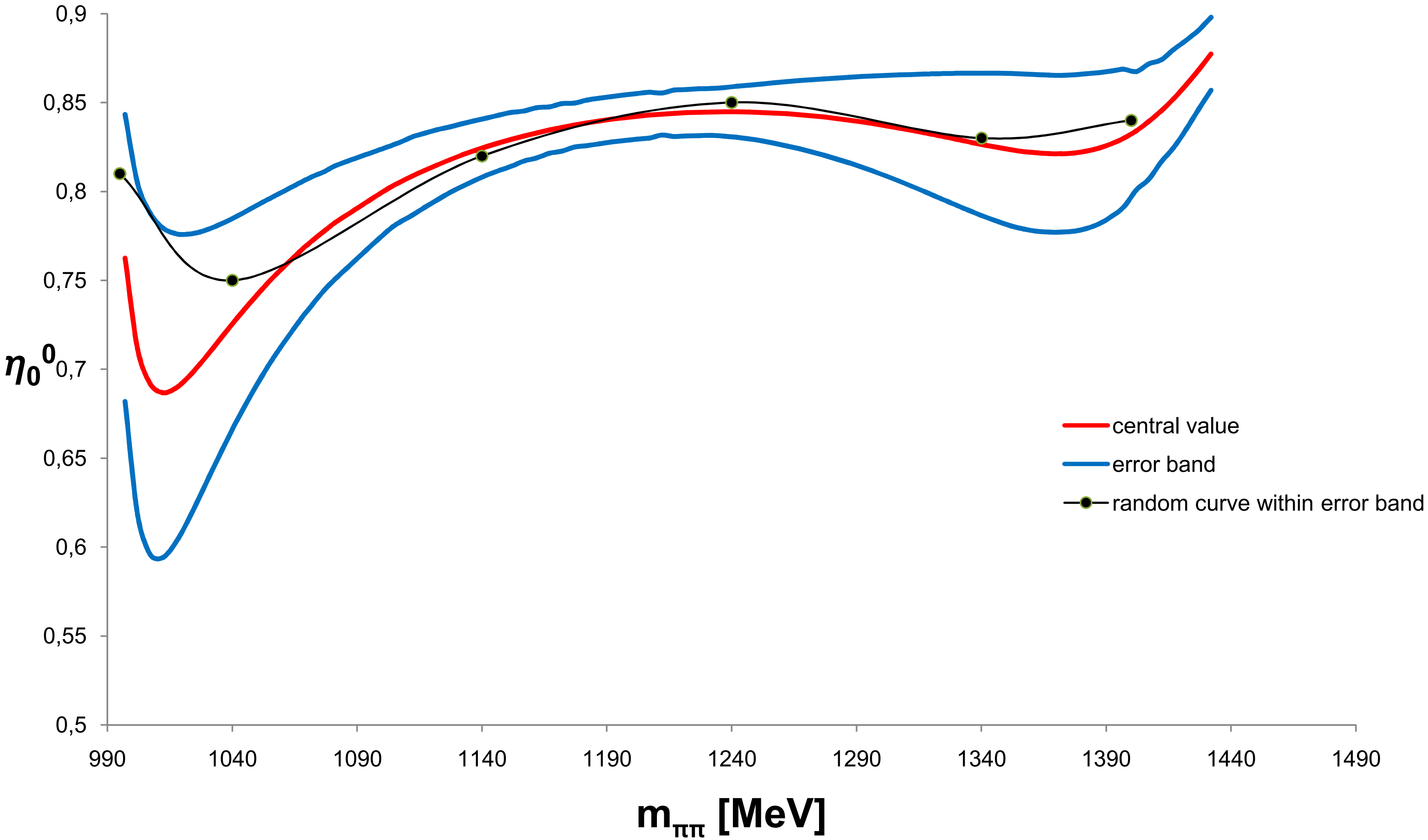
S0-wave

P-wave

S2-wave

FURTHER WORKS ON ERRORS

S0-wave inelasticies



CONCLUSIONS

1. Amplitudes fitted to Roy's equations also fulfill 1S ones.
2. First full analysis of errors of dispersion relations (Roy's and 1S equations) has been performed taking into account correlations between parameters in parameterizations.
3. Dispersion relations with one subtraction have bigger error near the pi-pi threshold and smaller near 1 GeV than those from Roy's equations.
4. 1S equations are very promising candidates for testing the crossing symmetry conditions of the S0, P and S2-waves up to about 1 GeV.

CONCLUSIONS

5. The input higher partial waves (D, F, G) in 1S equations are taken with higher weights than in Roy's equations.
6. It is worthy to work (and we do it) on direct analysis of the experimental data (independently on model parameterizations).
7. We hope that 1S dispersion relations will be widely used by all of us to test the pi-pi amplitudes parameterizations and even the row data.
8. One can derive 1S dispersion relations for higher partial waves.