Analysis of $B^{\pm} \to K^{+} K^{-} K^{\pm}$ decays

Collaboration :

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Motivation

1. study of rare B meson decays into three kaons,
2. search for direct CP violation,
3. partial wave analysis of the decay amplitudes,
4. constraints on final state interaction amplitudes,
5. investigation of the Dalitz diagrams,
6. description of the BaBar and Belle data,
7. application in hadron spectroscopy.

Decay amplitudes

Rare weak decays:

$$b \to \bar{u}us, \quad b \to \bar{d}ds \text{ and } b \to \bar{s}ss$$

Construction of the theoretical model:

1. application of QCD factorization in quasi-two body approach for a limited range of the effective K^+K^- masses less than 1.8 GeV,
2. description of final state K^+K^- interactions in terms of scalar and vector form factors,
3. introduction of unitary constraints in the S-wave coupled channel approach with the K^+K^- coupling to $\pi\,\pi$ and $4\,\pi$ (effective $\sigma\sigma$) states.

S wave amplitude in $B^- \to K_1^- K_2^+ K_3^-$

$$A_S = \tfrac{1}{2} G_F\, f_K (M_B^2 - m_{23}^2) F_0^{B^- \to (K^+K^-)_S}(m_K^2) U \chi \Gamma_2^{n*}(m_{23})$$
$$+ \tfrac{1}{2} G_F \frac{2\sqrt{2} B_0}{m_b - m_s} (M_B^2 - m_K^2) F_0^{B^- \to K^-}(m_{23}) V \Gamma_2^{s*}(m_{23})$$

$$U = \lambda_u [a_1 + a_4^u + a_{10}^u - (a_6^u + a_8^u r)] + \lambda_c [a_4^c + a_{10}^c - (a_6^c + a_8^c r)]$$

$$V = \lambda_u (-a_6^u + \tfrac{1}{2} a_8^u) + \lambda_c (-a_6^c + \tfrac{1}{2} a_8^c), \quad B_0 = m_\pi^2 / 2m_u$$

$$r = \frac{2m_K^2}{(m_b + m_u)(m_s + m_u)}; \quad \lambda_u = V_{ub} V_{us}^*, \quad \lambda_c = V_{cb} V_{cs}^*$$

Γ_2^n and Γ_2^s are scalar non-strange and strange kaon form factors.

P wave amplitude in $B^- \to K_1^- K_2^+ K_3^-$

$$\boxed{\begin{aligned} A_P &= 2\sqrt{2}\vec{p}_1 \cdot \vec{p}_2 G_F \{ \frac{f_K}{f_\rho} A_0^{B\to\rho}(m_K^2) F_u^{K^+K^-}(m_{23}) U \\ &- f_1^{BK}(m_{23})[F_u^{K^+K^-}(m_{23}) w_u + F_d^{K^+K^-}(m_{23}) w_d + F_s^{K^+K^-}(m_{23}) y]\} \end{aligned}}$$

$F_u^{K^+K^-}, F_d^{K^+K^-}, F_s^{K^+K^-}$ are vector kaon form factors.

$\vec{p}_1, \vec{p}_2$ are kaon K_1, K_2 momenta in the K_2, K_3 c.m. frame.

$$w_u = \lambda_u(a_2 + a_3 + a_5 + a_7 + a_9) + \lambda_c(a_3 + a_5 + a_7 + a_9)$$

$$w_d = \lambda_u[a_3 + a_5 - \tfrac{1}{2}(a_7 + a_9)] + \lambda_c[a_3 + a_5 - \tfrac{1}{2}(a_7 + a_9)]$$

$$y = \lambda_u[a_3 + a_4^u + a_5 - \tfrac{1}{2}(a_7 + a_9 + a_{10}^u)] + \lambda_c[a_3 + a_4^c + a_5 - \tfrac{1}{2}(a_7 + a_9 + a_{10}^c)]$$

Scalar non-strange and strange form factors Γ_2

$$\langle 0 | \bar{n}n | \overline{K}K \rangle = \sqrt{2} B_0 \Gamma_2^n(E)$$

$$\langle 0 | \bar{s}s | \overline{K}K \rangle = \sqrt{2} B_0 \Gamma_2^s(E)$$

$$\bar{n}n = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$

3 coupled channels: $\pi\pi, \overline{K}K, 4\pi(\sigma\sigma)$

$\boxed{\Gamma^* = R + TGR}$ ***R*** – **production** functions, ***T*** – matrix of **amplitudes**

$$\Gamma_2^{n,s*}(E) = R_2^{n,s}(E) + \sum_{j=1}^{3} R_j^{n,s}(E) \int \frac{d^3 p}{(2\pi)^3} \langle k_2 | T_{2j}(E) | p \rangle G_j(E,p) f_j(k_j,p)$$

$$G_j(E,p) = (E - 2\sqrt{p^2 + m_j^2} + i\varepsilon)^{-1}, \qquad f_j(k_j,p) = \frac{k_j^2 + \beta^2}{p^2 + \beta^2}$$

β - parameter

Multichannel model of the coupled amplitudes T is constrained by the data on pion-pion, kaon-antikaon and four pion production.

Chiral symmetry constraints

Low energy constraints of the kaon scalar form factors:

$$\boxed{\Gamma_{1,2}^{n,s} \approx a_{1,2}^{n,s} + b_{1,2}^{n,s} E^2, \qquad \Gamma_3 \approx 0, \qquad E \to 0.}$$

Parameters a and b are calculated using the chiral model of Meissner and Oller and the results of lattice QCD (from RBC and UKQCD Collaboration).

Parametrization of the **production functions**:

$$\boxed{R_j(E) = \frac{\alpha_j + \tau_j E + \omega_j E^2}{1 + CE^4}, \qquad j = 1,2,3}$$

Parametr C controls the high energy behaviour of R.

Analytical expressions for Γ's are obtained if separable potentials are used to describe the meson amplitudes T.

Unitarity conditions

$$\operatorname{Im}\Gamma_i^*(E) = \sum_{j=1}^{3} T_{ji}^*(E) r_j \Gamma_j^*(E)\theta(E-2m_j), \qquad i=1,2,3$$

$$r_j = -\frac{k_j E}{8\pi}, \quad k_j - \text{channel momenta}$$

Unitarity relations for the amplitudes :

$$\operatorname{Im} T_{ik}(E) = \sum_{j=1}^{3} T_{kj}^*(E) r_j T_{ij}(E)\theta(E-2m_j), \qquad i,k=1,2,3$$

Vector kaon form factors

Three vector form factors for q=u,d,s:

$$\langle K^+(p_1)K^-(p_2)|\bar{q}\gamma_\mu q|0\rangle = (p_1-p_2)_\mu F_q^{K^+K^-}(p_1-p_2)$$

Contributions to vector form factors from 8 vector mesons:

[ρ, ρ’, ρ’’] = [ρ(770), ρ(1450), ρ(1700)],

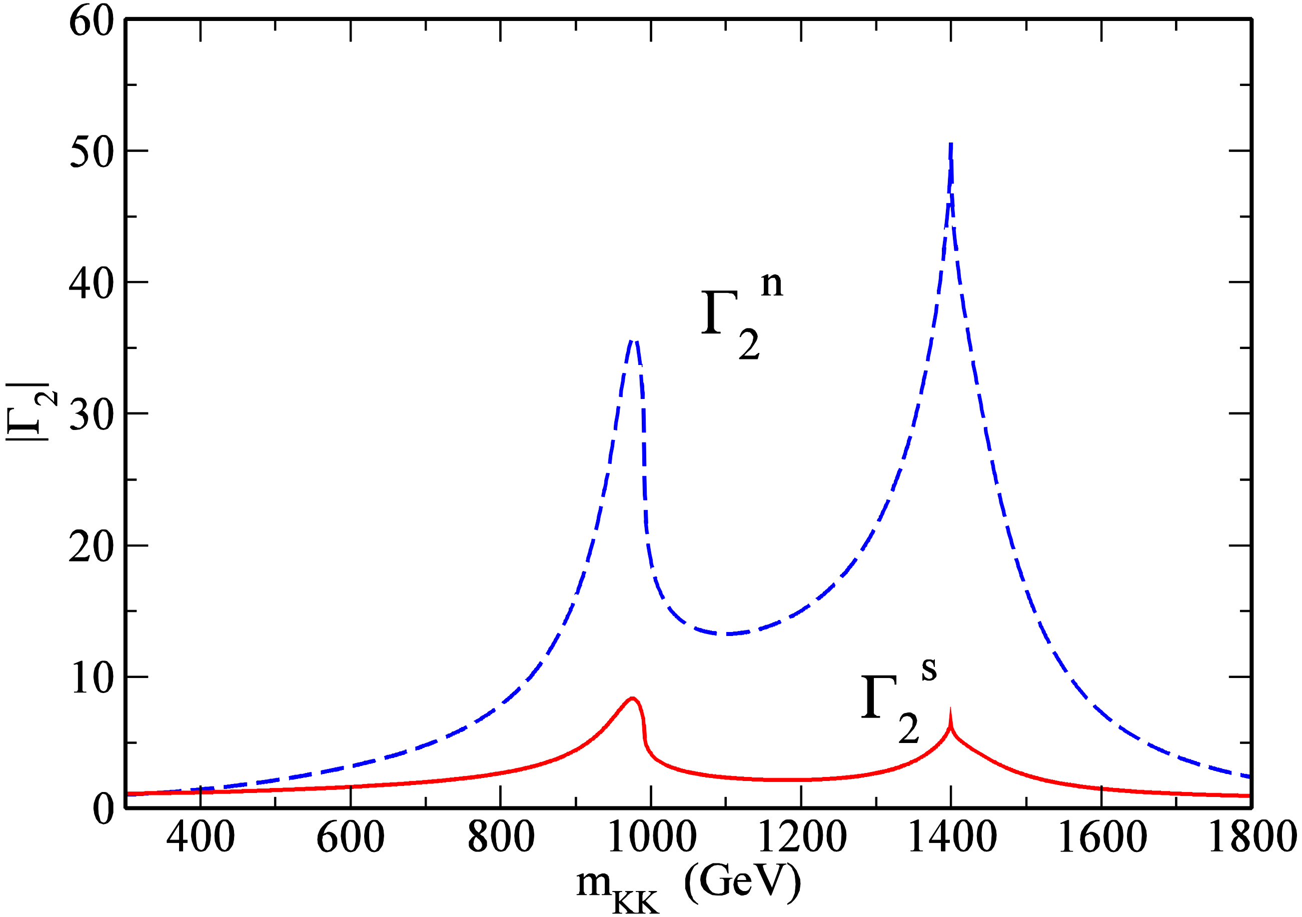
[ω, ω’, ω’’]= [ω(782), ω(1420), ω(1650)],

and [ϕ, ϕ’] = [ϕ(1020), ϕ(1680)].

$$\boxed{\begin{aligned} F_u^{K^+K^-} &= F_\rho + F_{\rho'} + F_{\rho''} + 3(F_\omega + F_{\omega'} + F_{\omega''}) \\ F_d^{K^+K^-} &= -(F_\rho + F_{\rho'} + F_{\rho''}) + 3(F_\omega + F_{\omega'} + F_{\omega''}) \\ F_s^{K^+K^-} &= -3(F_\phi + F_{\phi'}) \end{aligned}}$$

Components F_V are taken from the model of Bruch, Khodjamirian and Kuhn.

Scalar kaon form factors



Physical observables in $B^- \rightarrow K_1^- K_2^+ K_3^-$

1. Double effective mass and helicity angle branching fraction:

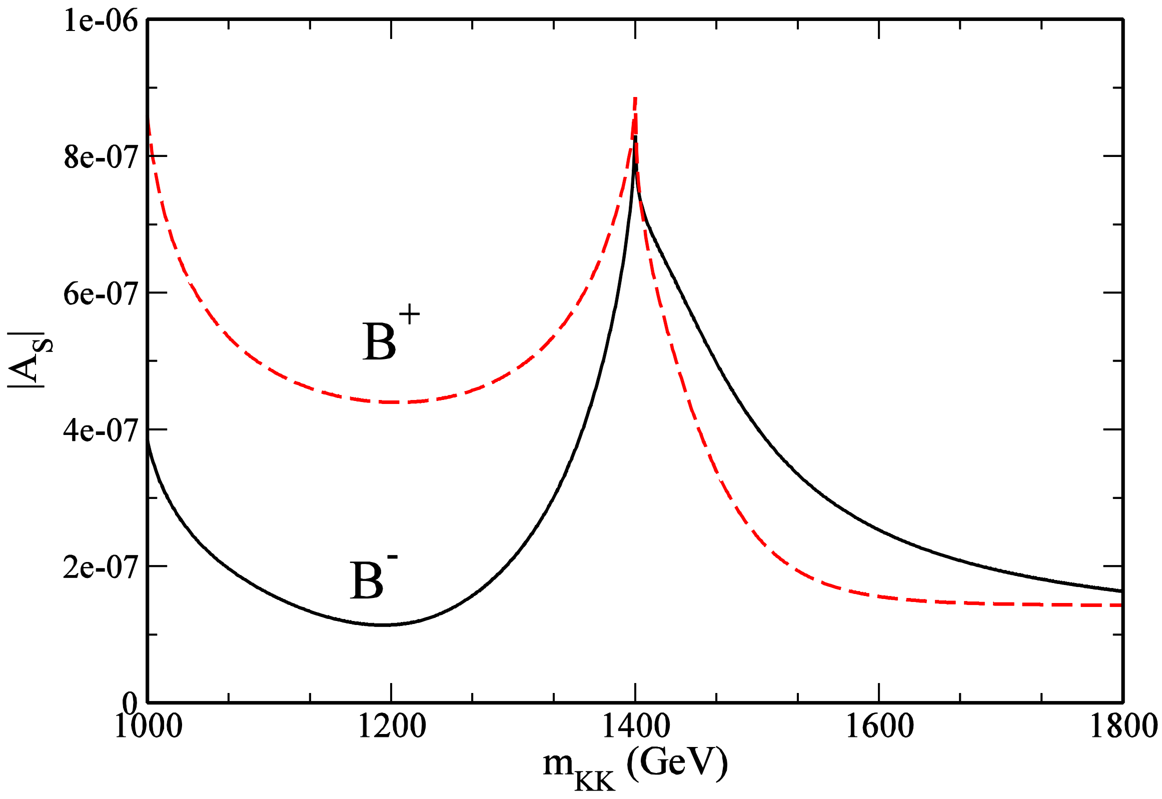
$$\boxed{\frac{dBr}{dm_{23} d\cos\theta_{12}} = PH\,|A|^2;}\quad PH = \frac{m_{23}\,|\vec{p}_1||\vec{p}_2|}{8(2\pi)^3 M_B{}^3 \Gamma_{B^-}}$$

$$A = \frac{1}{\sqrt{2}}[A_S(m_{23}) + A_P(m_{23})\cos\theta_{12} + \{1 \leftrightarrow 3\}]$$

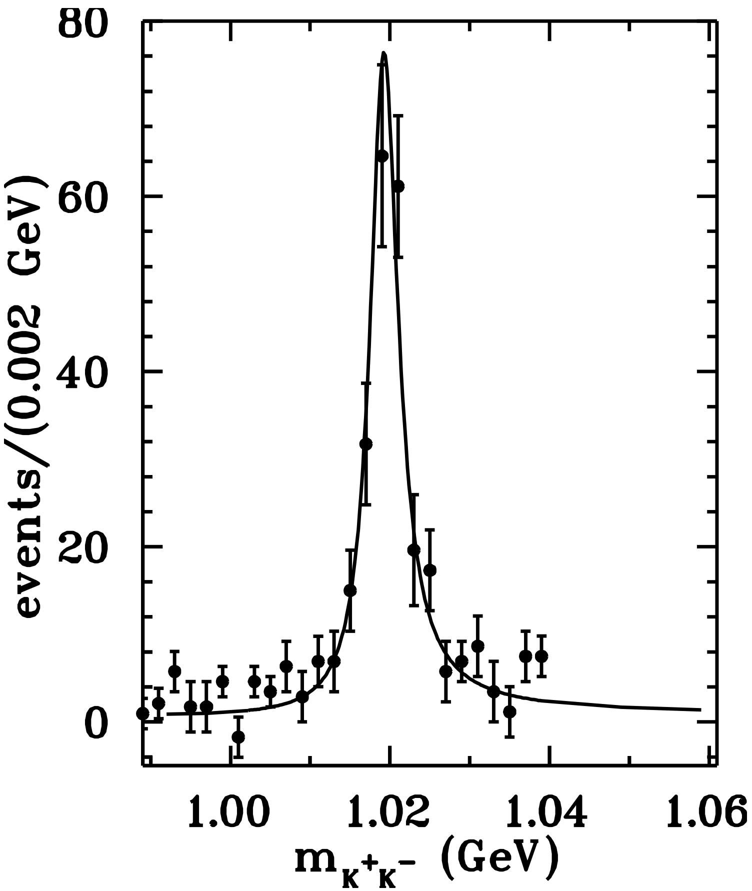
$$\cos\theta_{12} = \frac{\vec{p}_1 \cdot \vec{p}_2}{|\vec{p}_1 \cdot \vec{p}_2|}$$

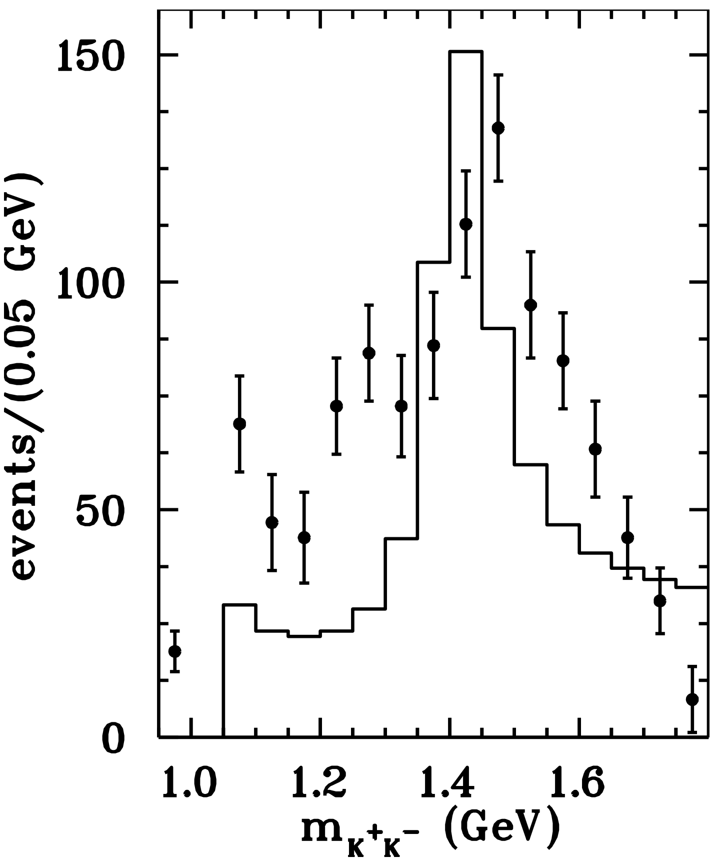
2. Effective mass m_{23} distributions
3. **Helicity angle** θ_{12} distributions

Example of scalar amplitudes



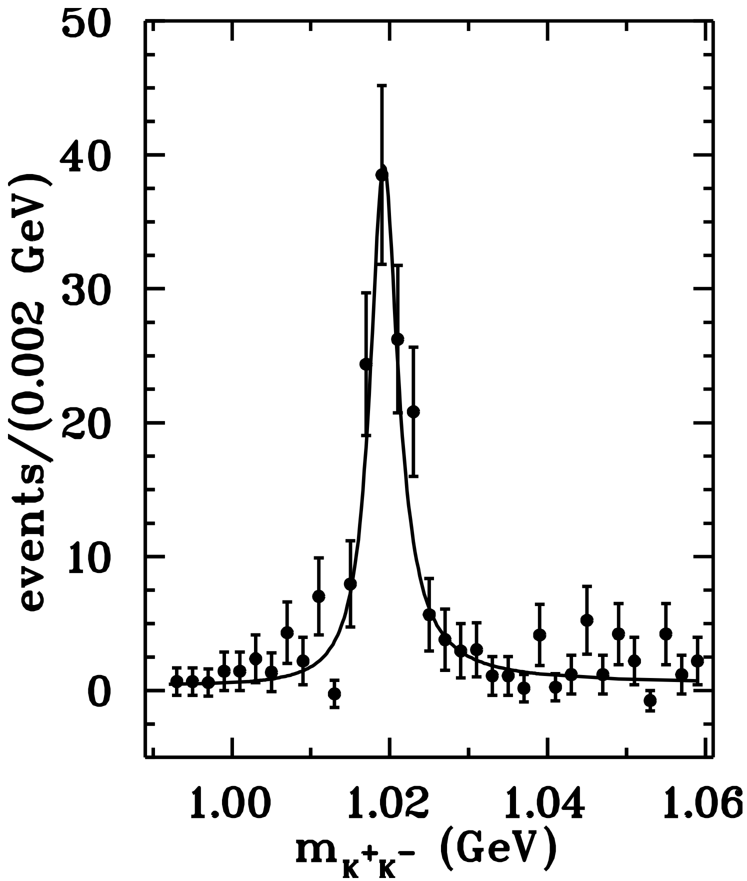
Comparison with the BaBar data: effective K^+K^- mass distribution

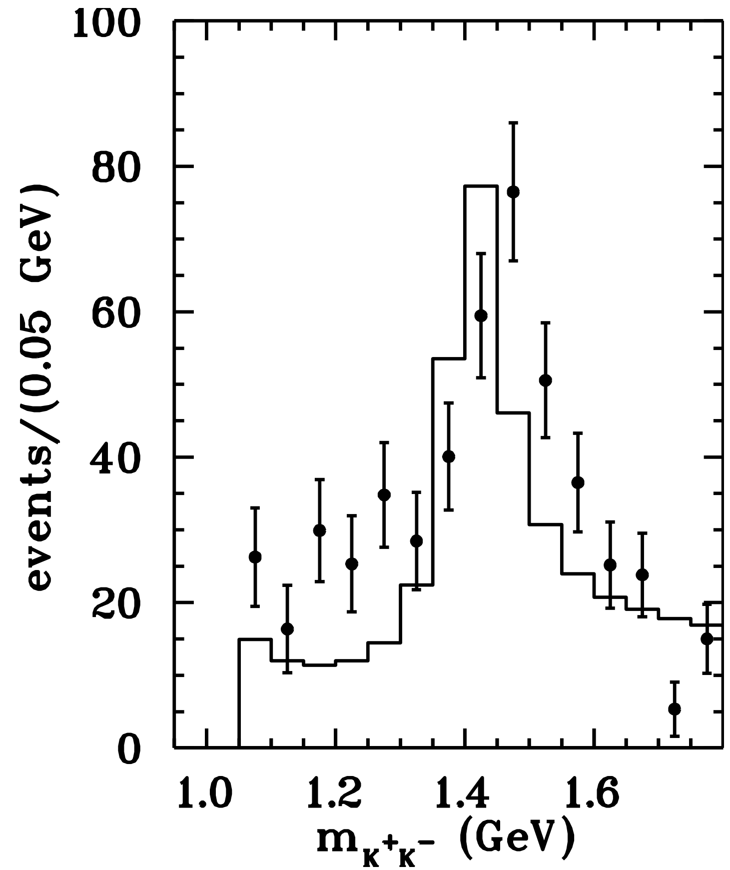




Data from BaBar Collaboration: Phys. Rev. D 74 (2006) 032003

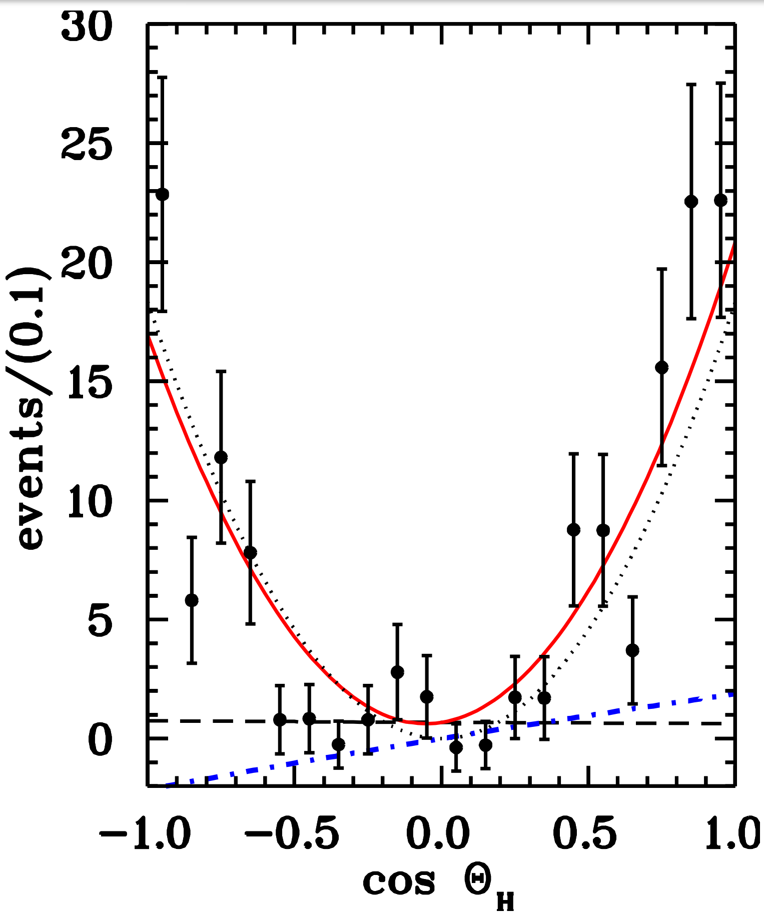
Comparison with the Belle data: effective K^+K^- mass distribution





Data from Belle Collaboration: Phys. Rev. D 71 (2005) 092003

Helicity angle distributions



········ P-wave, — — — — — S-wave, — · — · — · · interference

Data from Belle Colaboration: Phys. Rev. D 71 (2005) 092003, here $\cos\Theta_H = -\cos\Theta_{12}$.

Conclusions

1. Preliminary analysis of the $B^{\pm} \to K^+ K^- K^{\pm}$ decays has been performed with inclusion of the kaon-kaon strong interactions.
2. Final state $K^+ K^-$ interactions have been described using the **scalar strange and non-strange form factors** for the S-wave and the vector kaon form factor for the P-wave.
3. Good agreement of the three-body model with the Babar and Belle data is obtained, especially for the low $K^+ K^-$ masses.
4. The scalar resonance **f_0 (980)** leads to the **threshold enhancement** of the S-wave amplitude. The $K^+ K^-$ structure seen at 1.4 or 1.5 GeV can be attributed to another scalar resonance, coupled to the $K^+ K^-$, $\pi\pi$ and to the 4π system.
5. Sizable **helicity angle asymmetry** appears at the $K^+ K^-$ effective masses in the Φ range.
6. Potentially large CP asymmetry can be discovered in the part of the mass spectrum dominated by the **S**-wave .
7. New experimental analyses of data with **better statistics** are needed. The data already **exist!** For example, the Belle Collaboration has now five times larger data sample than that used in their first publication in 2005.