# Coupled-channels Faddeev calculation of $K^- d$ scattering length

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Antikaon-nuclear and atomic states – interesting exotic objects

Antikaon-nucleon interaction is the basic for their investigation

Questions/problems:

- Old or controversial experimental data on $K^- p$
- $\Lambda(1405)$ resonance question
  (bound state for $\bar{K}N$ and a resonance for $\pi\Sigma$? two resonances?)

It is not possible to give a preference to one- or two-pole structure due to imprecise two-body experimental data
=> use the two versions in three-body calculation.

SIDDHARTA experiment (kaonic deuterium atom):
the results of the experiment can be connected with the strong scattering length of $K^- d$ system

# Three-body coupled-channels equations

Faddeev equations in Alt - Grassberger - Sandhas form :

$$U_{11} = \qquad T_2 G_0 U_{21} + T_3 G_0 U_{31}$$

$$U_{21} = G_0^{-1} + T_1 G_0 U_{11} + T_3 G_0 U_{31}$$

$$U_{31} = G_0^{-1} + T_1 G_0 U_{11} + T_2 G_0 U_{21}$$

define unknown operators $U_{ij}$

$$U_{11}: \qquad 1 + (23) \rightarrow 1 + (23)$$

$$U_{21}: \qquad 1 + (23) \rightarrow 2 + (31)$$

$$U_{31}: \qquad 1 + (23) \rightarrow 3 + (12)$$

$\bar{K}N$ interaction strongly coupled with $\pi\Sigma$ via $\Lambda(1405)$ resonance

$\Rightarrow \pi\Sigma$ channel included directly. Particle channels $(\alpha)$ :

$$\alpha = 1 : \left| \bar{K}_1 \; N_2 \; N_3 \right\rangle, \quad \alpha = 2 : \left| \pi_1 \; \Sigma_2 \; N_3 \right\rangle, \quad \alpha = 3 : \left| \pi_1 \; N_2 \; \Sigma_3 \right\rangle$$

| |
|---|
| $i, j$ - usual Faddeev indexes |
| $\alpha, \beta$ - channel indexes |

Two-body $T$-matrices, $T_i^{\alpha\beta}$ :

$$T_1 = \begin{pmatrix} T_1^{NN} & 0 & 0 \\ 0 & T_1^{\Sigma N} & 0 \\ 0 & 0 & T_1^{\Sigma N} \end{pmatrix}, \; T_2 = \begin{pmatrix} T_2^{KK} & 0 & T_2^{K\pi} \\ 0 & T_2^{\pi N} & 0 \\ T_2^{\pi K} & 0 & T_2^{\pi\pi} \end{pmatrix}, \; T_3 = \begin{pmatrix} T_3^{KK} & T_3^{K\pi} & 0 \\ T_3^{\pi K} & T_3^{\pi\pi} & 0 \\ 0 & 0 & T_3^{\pi N} \end{pmatrix}$$

$T^{NN}$, $T^{\Sigma N}$, and $T^{\pi N}$ are one-channel (usual) $T$-matrices;

$T^{KK}$ : $\bar{K}N \to \bar{K}N$, $\quad T^{K\pi}$ : $\pi\Sigma \to \bar{K}N$,

$T^{\pi K}$ : $\bar{K}N \to \pi\Sigma$, $\quad T^{\pi\pi}$ : $\pi\Sigma \to \pi\Sigma$ $\qquad$ - elements of 2-channel $T^{\bar{K}N-\pi\Sigma}$

Free Green functions $G_0^{\alpha\beta} = \delta_{\alpha\beta}\, G_0^{\alpha}$, transition operators $U_{ij}^{\alpha\beta}$

Quantum numbers of $\bar{K}NN - \pi\Sigma N$ system ($K^- d$):

spin $S = 1$, orbital momentum $L = 0$, isospin $I = 1/2$

Two identical nucleons - antisymmetrization,

finally: system of 10 integral equations (two-term $NN$ potential)

## Coupled-channels $\bar{K}N - \pi\Sigma$ interaction:

*J. Révai, N.V. Shevchenko, Phys. Rev. C 79 (2009) 035202; new fits*

$\bar{K}$N is strongly coupled with $\pi\Sigma$ channel through $\Lambda(1405)$ resonance

$$\text{PDG:}\ \ E_\Lambda = 1406.5 - i\ 25.0\ \text{MeV},\ \ I = 0$$

Usual assuption:

a resonance in $I = 0$ $\pi\Sigma$ and a quasi-bound state in $I = 0$ $\bar{K}N$ channel

Alternative version: $\Lambda(1405)$ is an effect of two close poles

=> phenomenological potential with one- and two-pole structure of $\Lambda(1405)$ resonance.

*Isospin-breaking effects*:

1. Direct inclusion of *Coulomb interaction* (kaonic hydrogen)
2. Using of the *physical masses*: $m_{K^-}, m_{\bar{K}^0}, m_p, m_n$ instead of $m_{\bar{K}}, m_N$

Existing experimental data:

- Scattering data:

  - Cross-sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions,
  - Threshold branching ratios $\gamma$, $R_c$, and $R_n$

    *D.N.Tovee et al., Nucl. Phys. B*33 (1971) 493,

    *R.J. Nowak et al., Nucl. Phys. B*139 (1978) 61

  Without directly included $\pi\Lambda$ channel: $R_{\pi\Sigma} = \dfrac{R_c}{1 - R_n(1 - R_c)}$

- Strong interaction shift and width of the $K^-p$ atom 1*s* level state

  KEK (*M. Iwasaki et al.,* Phys. Rev. Lett. 78 (1997) 3067):

  $$\Delta E_{1s}^{KEK} = -323 \pm 63 \pm 11 \text{ eV}, \quad \Gamma_{1s}^{KEK} = 407 \pm 208 \pm 100 \text{ eV}$$

  DEAR (*G. Beer et al.,* Phys. Rev. Lett. 94 (2005) 212302):

  $$\Delta E_{1s}^{DEAR} = -193 \pm 37 \pm 6 \text{ eV}, \quad \Gamma_{1s}^{DEAR} = 249 \pm 111 \pm 30 \text{ eV}$$

Strong part of the total potential $V_s + V_C$ :

$$V_I^{\alpha\beta}(\vec{k}^{\alpha}, \vec{k}'^{\beta}) = g_I^{\alpha}(\vec{k}^{\alpha})\, \lambda_{\mathrm{I}}^{\alpha\beta}\; g_I^{\beta}(\vec{k}'^{\beta}),$$

$\alpha, \beta = K(\bar{K}N$ channel) or $\pi(\pi\Sigma$ channel); $I = 0$ or $1$;

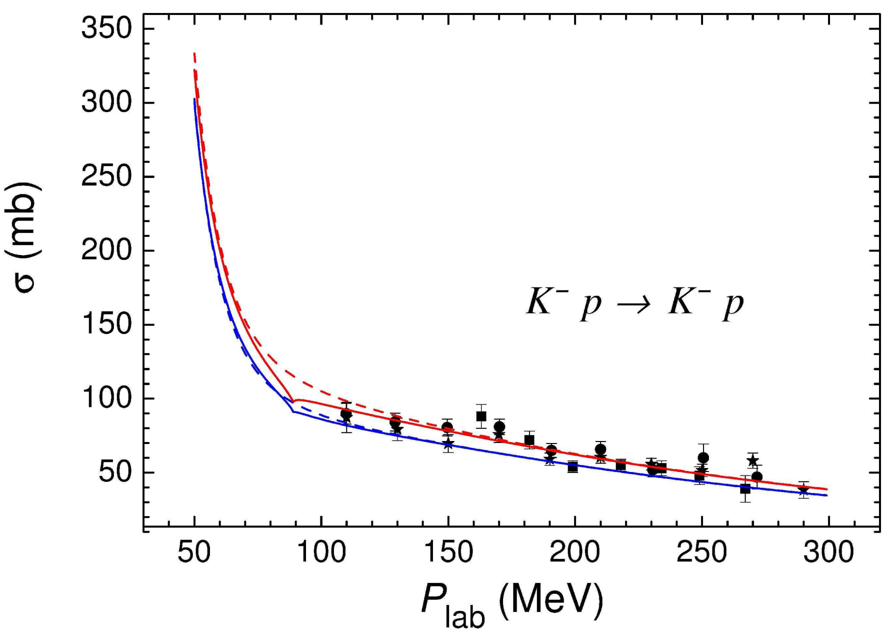
• 1 - pole $\Lambda(1405)$ :

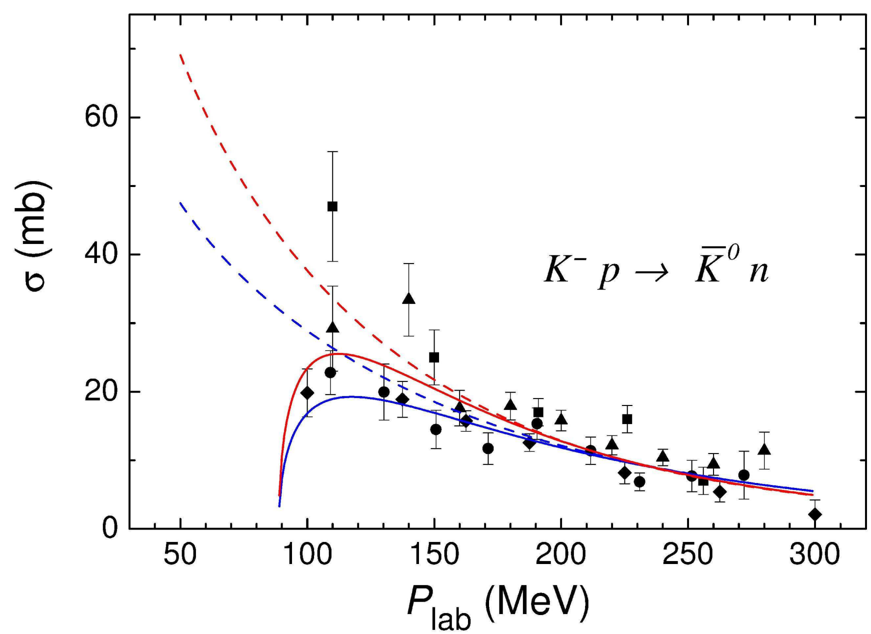
$$g_{I,1pole}^{\alpha}(k^{\alpha}) = \frac{1}{(k^{\alpha})^2 + (\beta_{\mathrm{I}}^{\alpha})^2} \qquad \text{for } \alpha = K \text{ or } \pi$$

• 2 - pole $\Lambda(1405)$ :

$$g_{I,1pole}^{\alpha}(k^{\alpha}) = \frac{1}{(k^{\alpha})^2 + (\beta_{\mathrm{I}}^{\alpha})^2} \qquad \text{for } \alpha = K$$

$$g_{I,2pole}^{\alpha}(k^{\alpha}) = \frac{1}{(k^{\alpha})^2 + (\beta_{\mathrm{I}}^{\alpha})^2} + \frac{s\,(\beta_{\mathrm{I}}^{\alpha})^2}{[\,(k^{\alpha})^2 + (\beta_{\mathrm{I}}^{\alpha})^2\,]^2} \qquad \text{for } \alpha = \pi$$

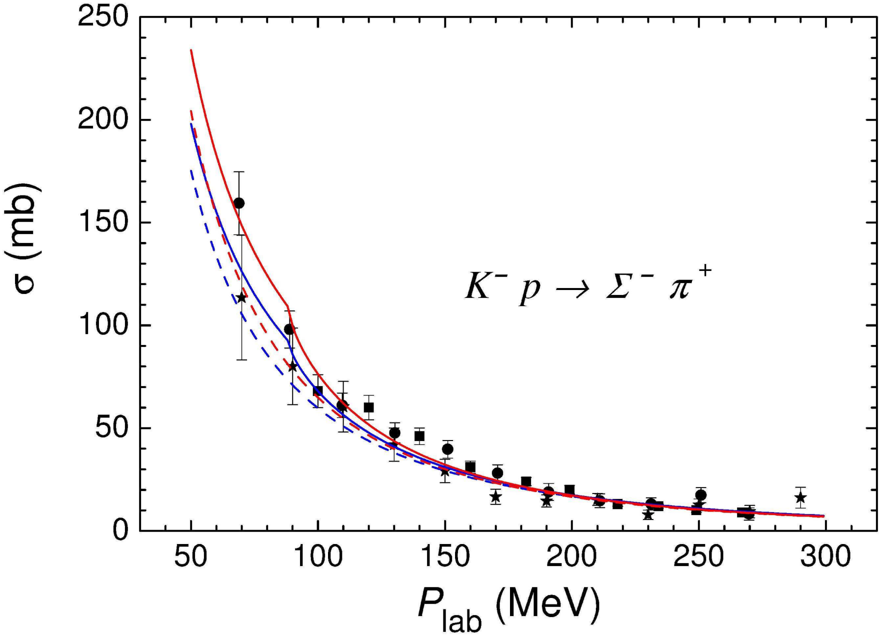


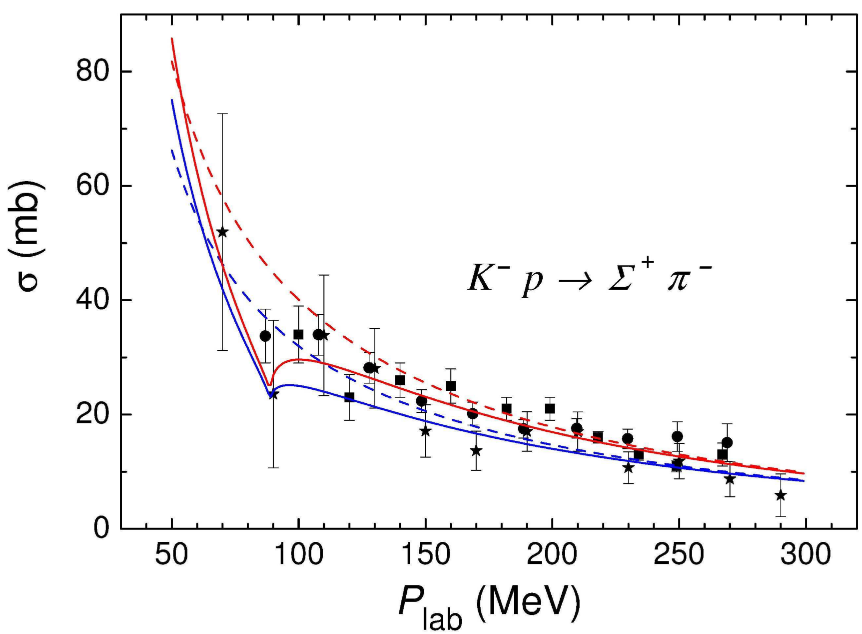



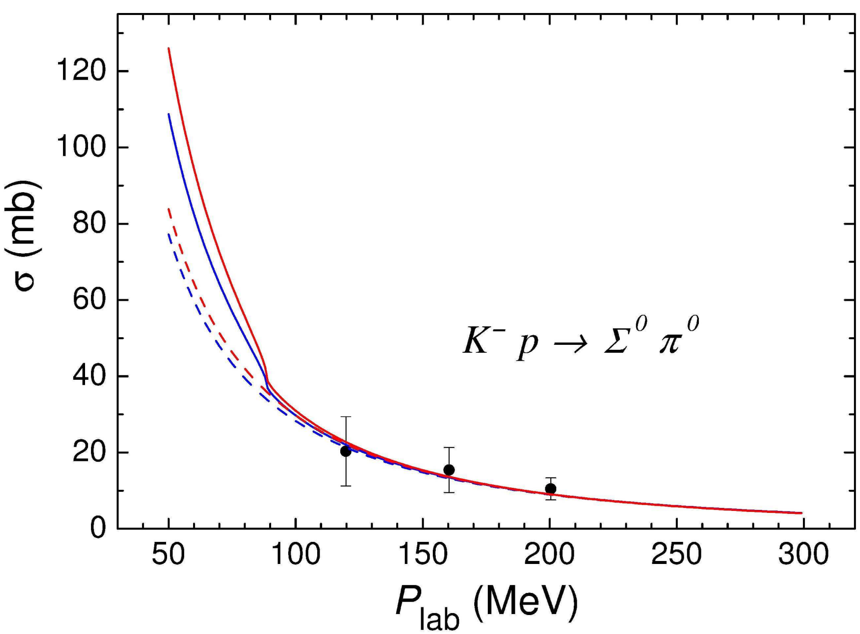

<u>Comparison with experimental data, cross-sections:</u>

one-pole potential: with physical masses (blue solid line)
with averaged masses (blue dashed line)

two-pole potential: with physical masses (red solid line)
with averaged masses (red dashed line)

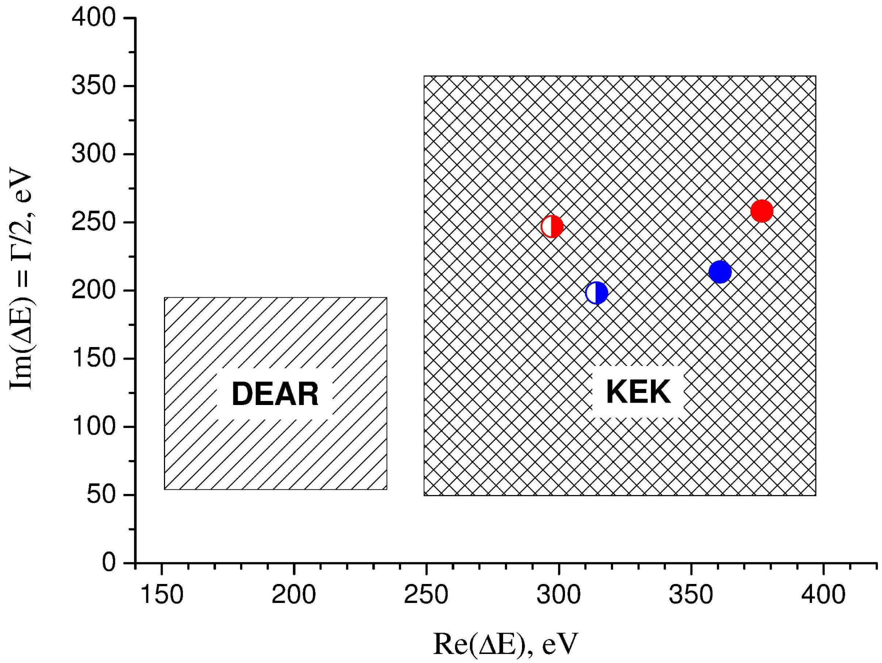



<u>Comparison with experimental data</u>
(continuation)

## Experimental and theoretical 1s $K^-p$ level shift and width:




One-pole potential:
physical masses (blue filled circle)
averaged masses (blue half-empty circle)

Two-pole potential:
physical masses (red filled circle)
averaged masses (red half-empty circle)

## Pole positions

| "1-pole" | "2-pole" |
|---|---|
| 1414 – *i* 50 MeV | 1412 – *i* 33 MeV<br>1380 – *i* 105 MeV |

# Two-term $NN$ potential

*P. Doleschall, private communication, 2009*

$$V_{NN} = \sum_{i=1}^{2} \left| g_i \right\rangle \lambda_i \left\langle g_i \right| \quad \rightarrow$$

$$T_{NN} = \sum_{i,j=1}^{2} \left| g_i \right\rangle \tau_{ij} \left\langle g_j \right|$$

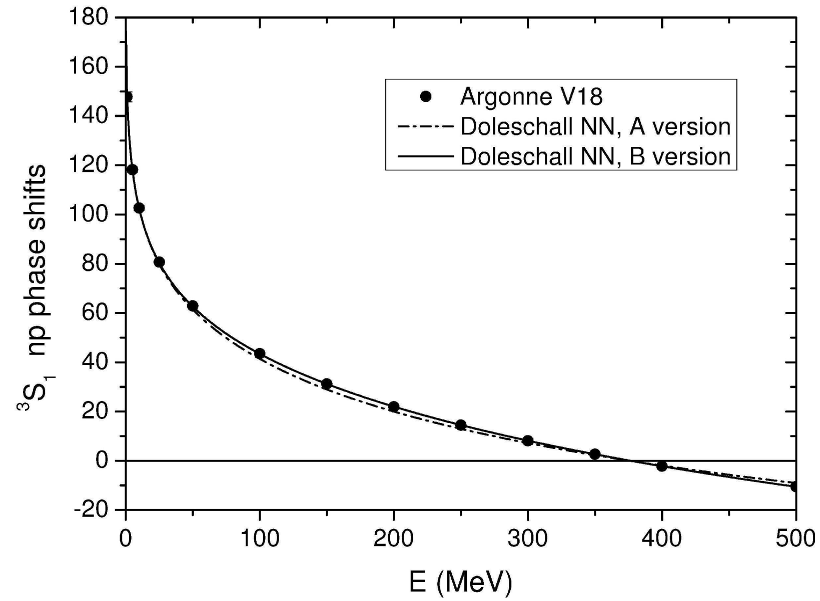
Reproduces:

Argonne V18 NN phase shifts (with sign change!),

$$a^A(np) = -5.402 \text{ fm}, \; r_{eff}^A(np) = 1.754 \text{ fm},$$

$$a^B(np) = -5.413 \text{ fm}, \; r_{eff}^B(np) = 1.760 \text{ fm},$$

and $E_{deu} = -2.2246$ MeV.




$$\text{Version A}: \quad g_i^A(k) = \sum_{m=1}^{2} \frac{\gamma_{im}^A}{\left(\beta_{im}^A\right)^2 + k^2}, \; i = 1, 2$$

$$\text{Version B}: \quad g_1^B(k) = \sum_{m=1}^{3} \frac{\gamma_{1m}^B}{\left(\beta_{1m}^B\right)^2 + k^2}, \; g_2^B(k) = \sum_{m=1}^{2} \frac{\gamma_{2m}^B}{\left(\beta_{2m}^B\right)^2 + k^2}$$

# PEST *NN* potential

*H. Zankel, W. Plessas, and J. Haidenbauer, Phys. Rev. C28 (1983) 538*

Separable isospin-dependent $T$-matrices: $T_{i,I}^{\alpha\beta} = \left| g_{i,I}^{\alpha} \right\rangle \tau_{i,\mathrm{I}}^{\alpha\beta} \left\langle g_{i,I}^{\beta} \right|$

$\underline{T_I^{NN}(k,k';z)}$ corresponds to

$$V_I^{NN}(k,k') = -g_I^{NN}(k)\, g_I^{NN}(k') \qquad \text{with} \qquad g_I^{NN}(k) = \frac{1}{2\sqrt{\pi}} \sum_{i=1}^{6} \frac{c_{i,I}^{NN}}{k^2 + \left(\beta_{i,I}^{NN}\right)^2}$$

A separabelization of a Paris potential; $I = 0$ or 1

Gives:

$$E_{deuteron} = -2.2249 \text{ MeV},$$

$$a\left({}^3S_1\right) = -5.422 \text{ fm},$$

$$a\left({}^1S_0\right) = 17.534 \text{ fm},$$

# $\Sigma N(-\Lambda N)$ interaction

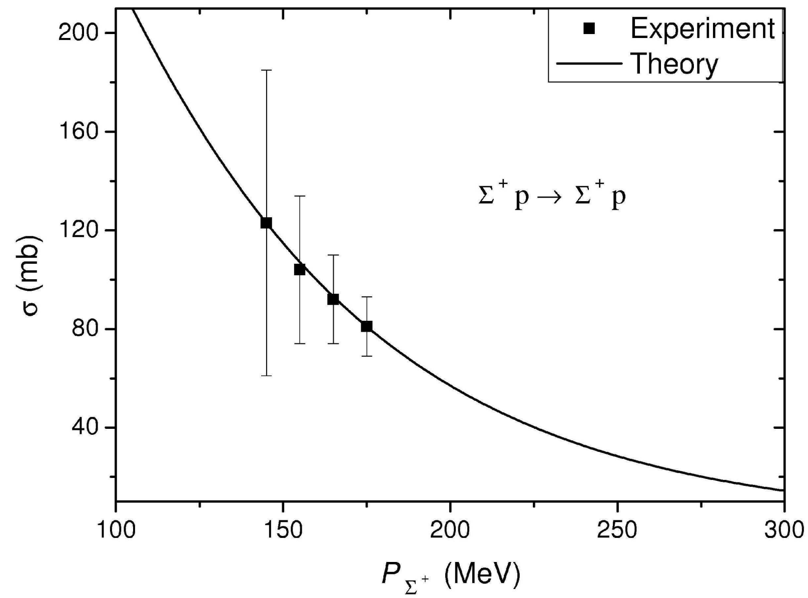
*J. Révai, N.V. Shevchenko, 2009*

$\mathrm{T}_I^{\Sigma N}(k,k';z)$ corresponds to

$$\mathrm{V}_I^{\Sigma N}(k,k') = \lambda_I^{\Sigma N}\; g_I^{\Sigma N}(k)\; g_I^{\Sigma N}(k')$$

$$\text{with}\;\; g_I^{\Sigma N}(k) = \frac{1}{k^2 + \left(\beta_I^{\Sigma N}\right)^2}$$

All parameters were fitted to reproduce experimental cross-sections



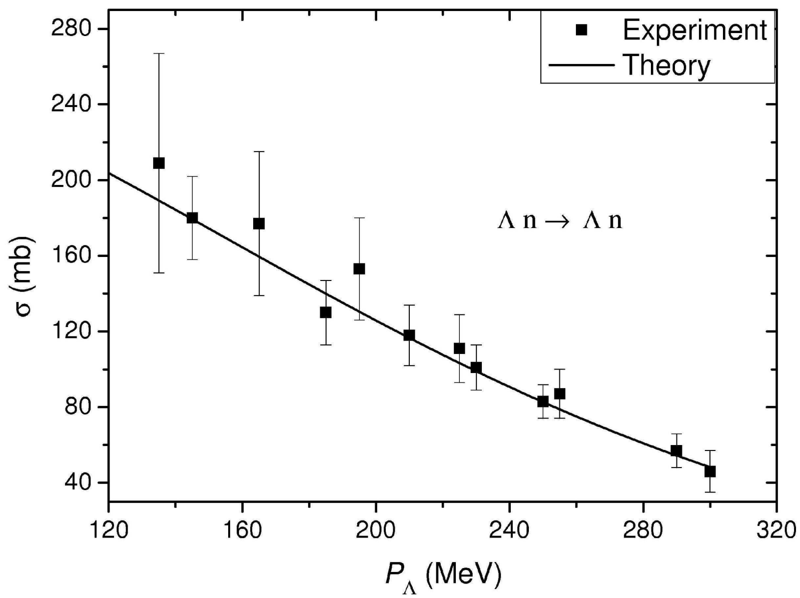

Pure *I=3/2* part

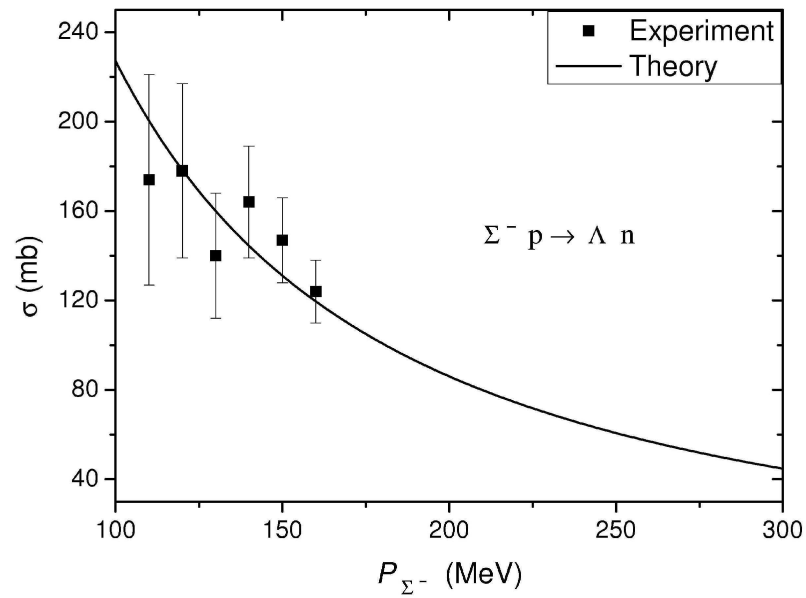
*I=3/2*

Real parameters, one-channel case

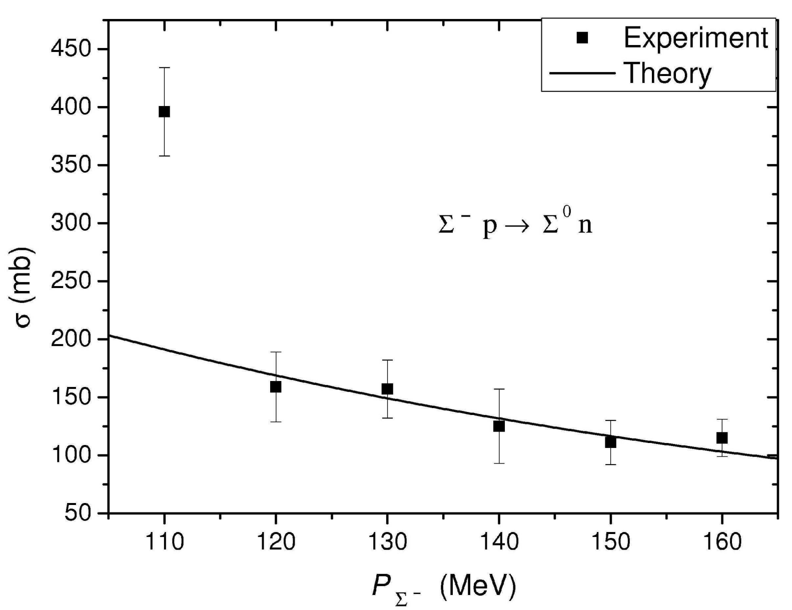
*I=1/2*

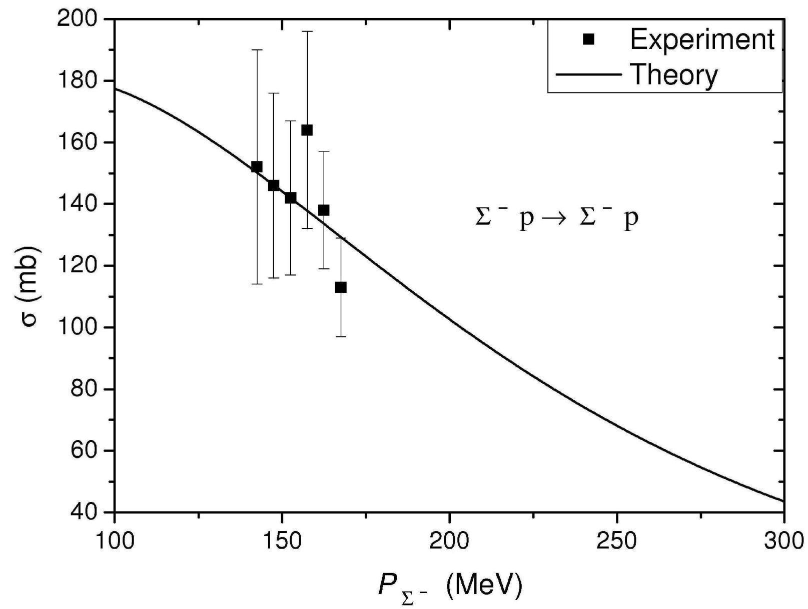
1. Two-channel $\Sigma N - \Lambda N$ potential, real parameters
2. One-channel $\Sigma N$ potential, complex strength parameter



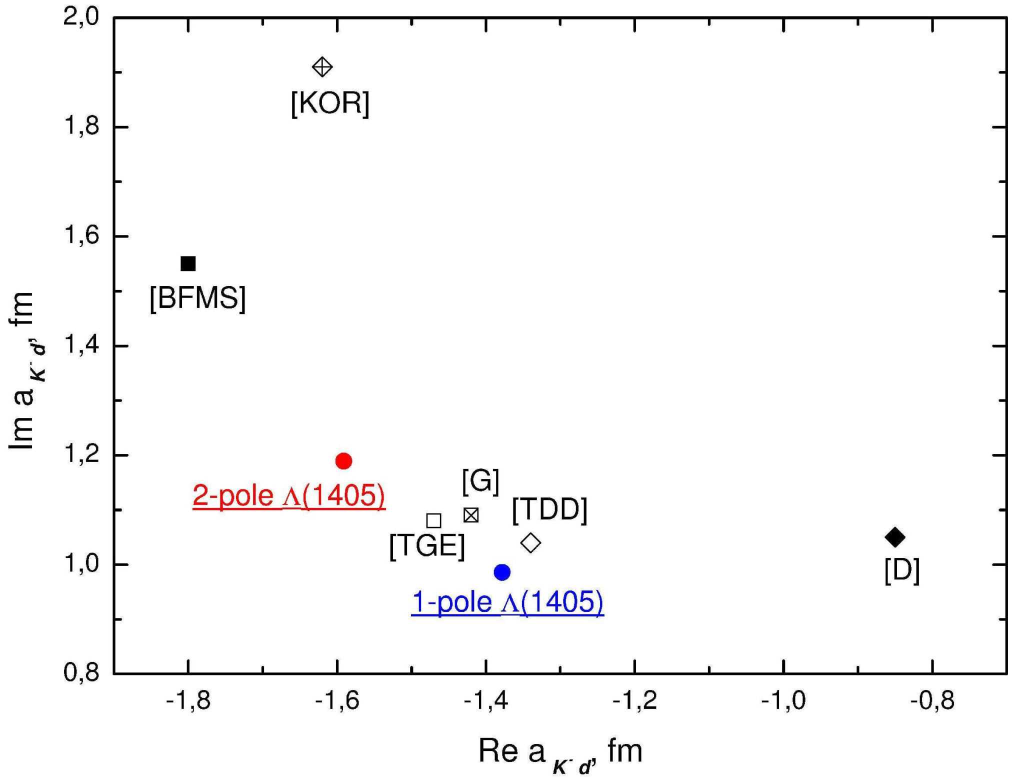




Pure *I=1/2* and *I=1/2*, *I=3/2* mixtures



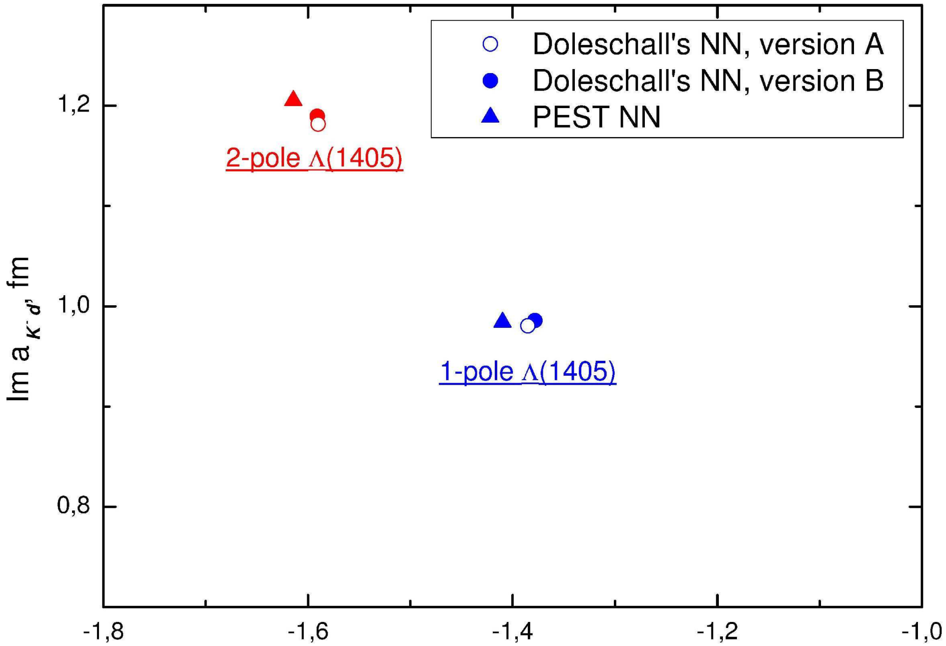



# Results

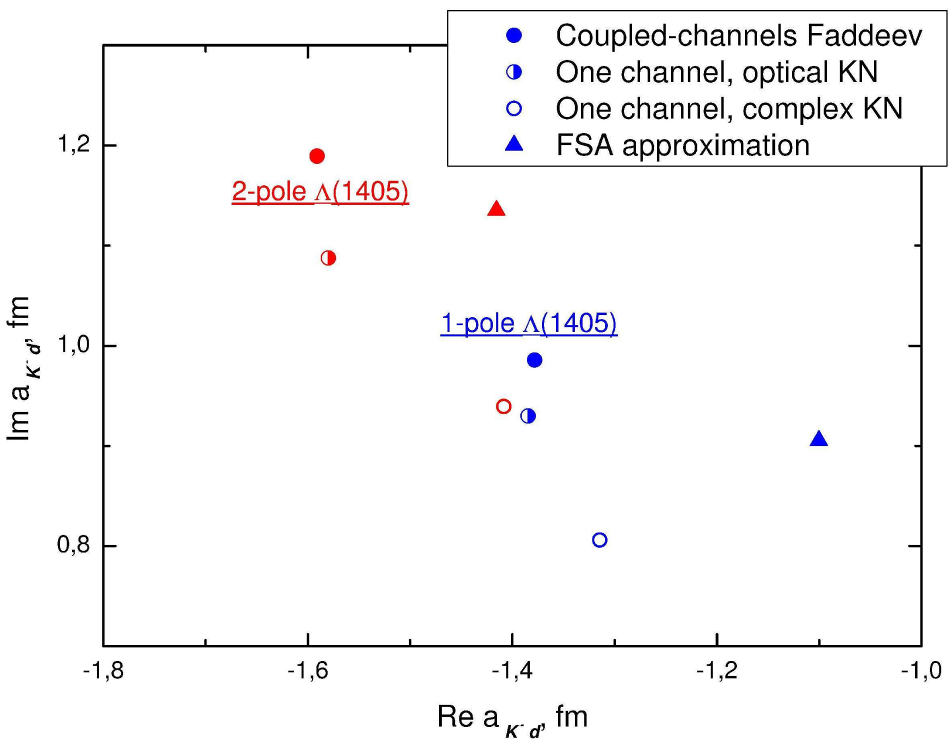



Comparison with other calculations.

Full calculation with 1-pole (blue circle) and 2-pole (red circle) version of $\bar{K}N-\pi\Sigma$ interaction. Doleschall's *NN*, version B is used.




Dependence of the results on *NN* interaction:
1-pole (blue) and 2-pole (red) versions of $\bar{K}N - \pi\Sigma$ interaction




Comparison with approximated results:
one-channel Faddeev with optical, complex $\bar{K}N$; and FSA (Fixed Scatterer Approximation) results.

# Conclusions:

- Coupled-channels Faddeev-type (AGS) calculation of
$K^- d$ scattering length with one- and two-pole $\bar{K}N - \pi\Sigma$ :

$$a_{1-pole}(K^- d) = -1.38 + i\ 0.99 \ \text{fm},$$

$$a_{2-pole}(K^- d) = -1.59 + i\ 1.19 \ \text{fm}.$$

- Dependence of the results on the $NN$ potential is weak

- Calculations with commonly used approximations:
  - one-channel Faddeev calculation with optical $\bar{K}N$ potential,
  - one-channel Faddeev calculation with complex $\bar{K}N$ potential,
  - Fixed Scatterer Approximation.

FSA (“FCA”) is improper for $K^- d$ system