Spectral-Statistics Analysis of the Light Meson Spectrum

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in collaboration with
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Meson Spectrum

- Quark-antiquark states
- States that go beyond the quark-antiquark picture
    - Exotics
    - Hybrids
    - Glueballs
- Theoretical input is needed to understand the spectrum

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Suggestion:

Look at the statistical properties of the spectrum

Spectral Fluctuations and the Underlying Dynamics

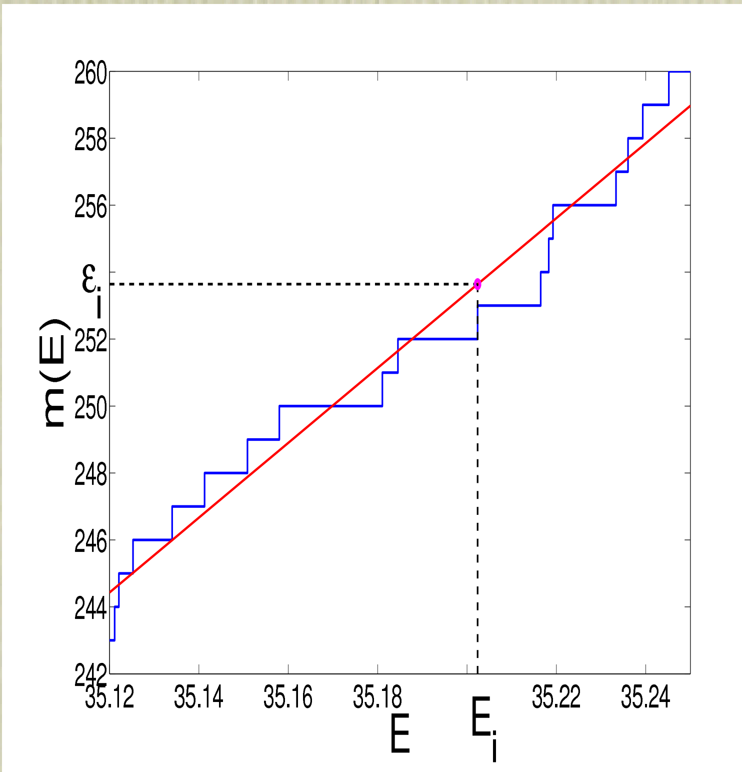
- The energy spectrum is fully characterized by its level density
- It can be split into a smooth part $\bar{\rho}(E)$, giving the secular behavior with the energy, and a fluctuating part $\tilde{\rho}(E)$, responsible for the statistical properties of the spectrum
- The process of removing the smooth part is called unfolding
- Three possible scenarios:
  - Integrable (uncorrelated): Fluctuations follow a Poisson distribution
  - Chaotic (fully correlated): Fluctuations follow a Wigner surmise
  - Intermediate: Berry-Robnik

$$f_{BR} = \alpha f_W + (1-\alpha) f_P$$

Unfolding

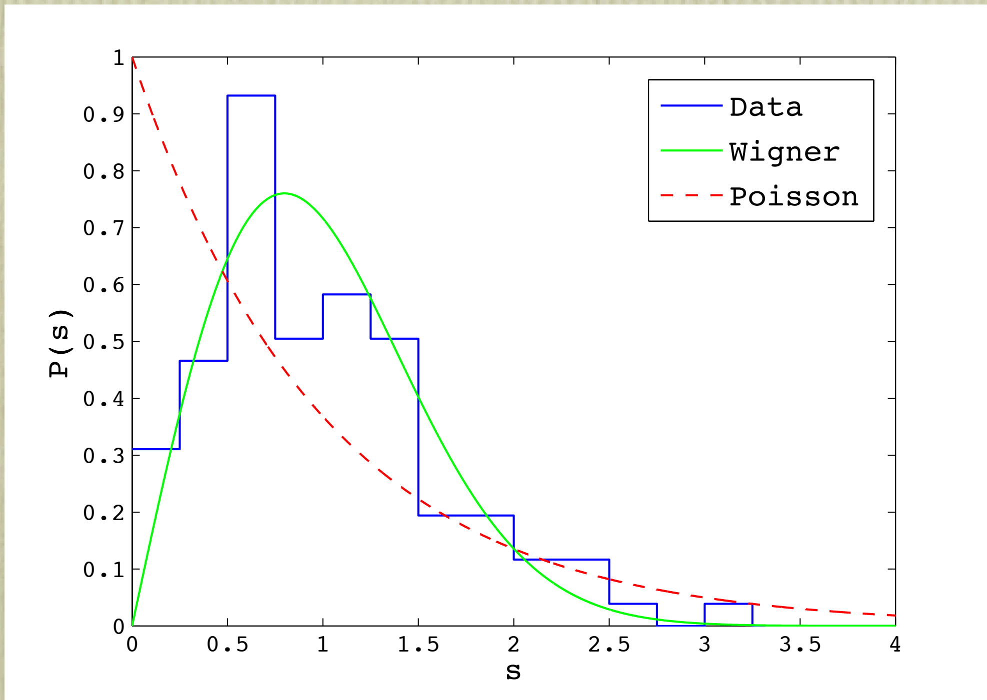
Level density: $\rho(E) = \hat{\rho}(E) + \tilde{\rho}(E)$

Accumulated level density: $m(E) = \int_{-\infty}^{E} \rho(x)dx = \bar{m}(E) + \tilde{m}(E)$



Data vs Wigner & Poisson

$$s_i = \frac{S_i}{\langle S \rangle} \quad ; \quad \langle S \rangle = (l_x - 1)^{-1} \sum_{i=1}^{l_x - 1} S_i \quad ; \quad S_i = E_{i+1} - E_i$$



$$P_P(s) = \exp[-s]$$

$$P_W = (s) = \frac{\pi s}{2} \exp\left[-\frac{\pi s^2}{4}\right]$$

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- Low statistics: Distorted distributions
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- Momenta

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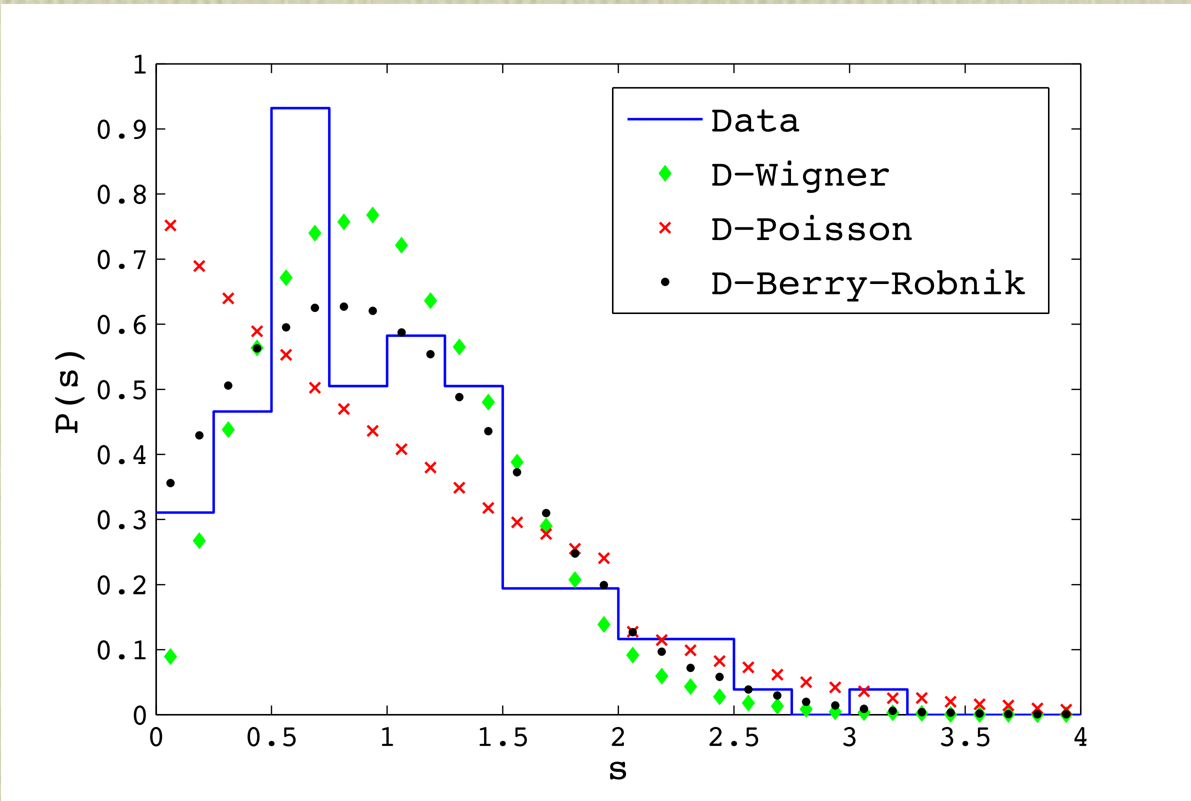

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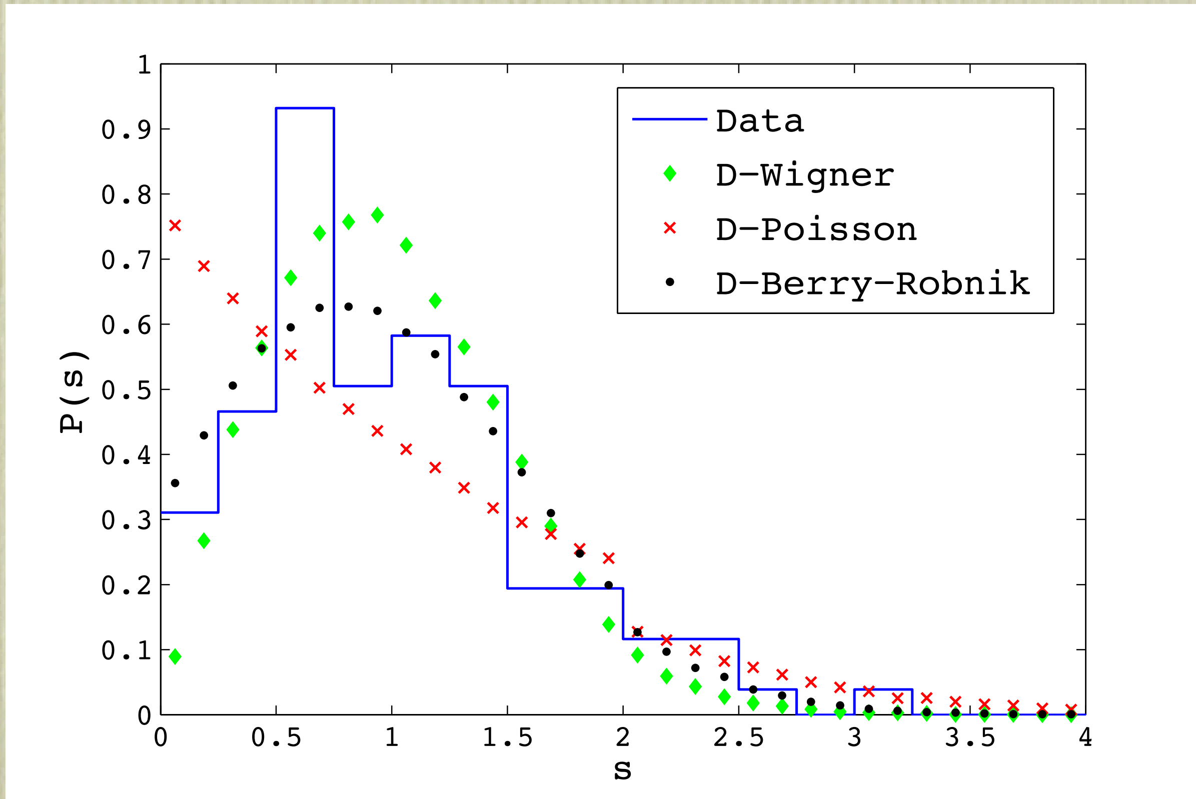


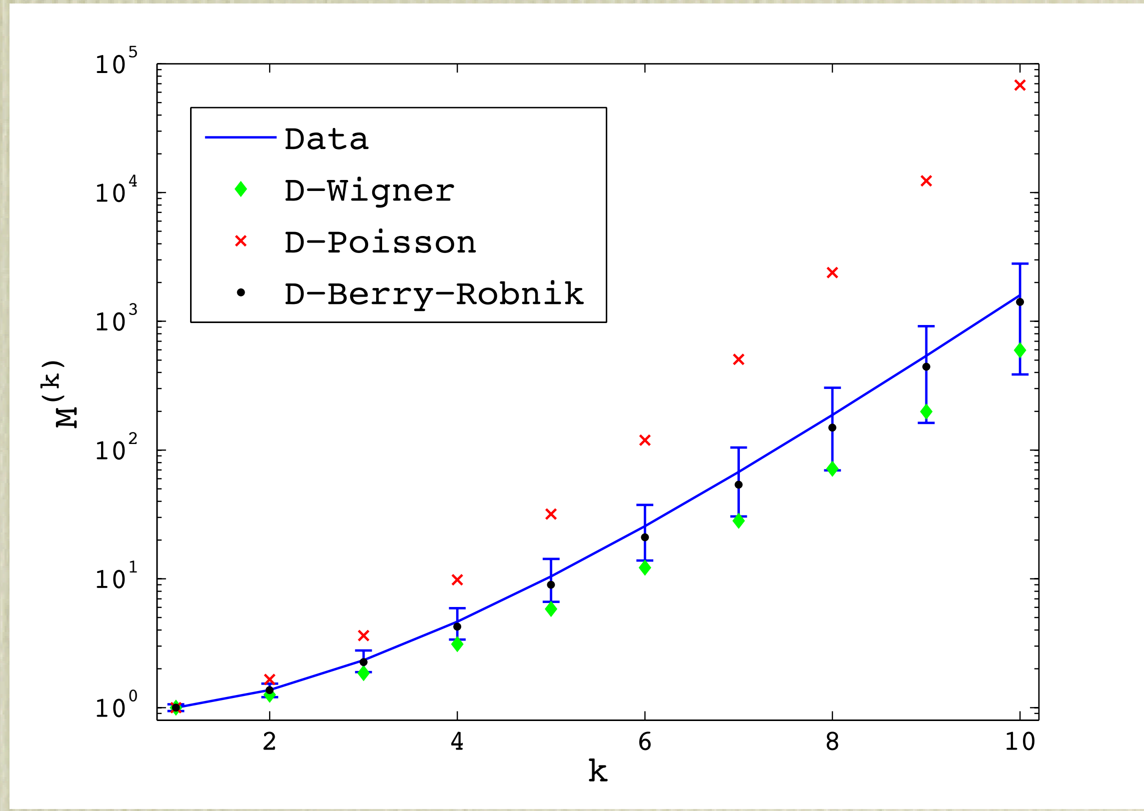
pDW	0.47
pDP	0.13
pDBR (78% of chaos)	0.68

- Data from PDG: Nakamura *et al.*, *JPG* 37 (2010) 075021, and 2011 partial update for the 2012 edition

- Muñoz, Fernández-Ramírez, Relaño, Retamosa, *PLB* 710 (2012) 139

Momenta

$$M^{(k)} = (d-n)^{-1} \sum_{i=1}^{d-n} s_i^k$$



Theoretical Spectra

- Constituent Quark Models
  - Godfrey, Isgur, *PRD* 32 (1985) 189
  - Koll, *et al.*, *EPJA* 9 (2000) 73
  - Vijande, *et al.*, *JPG* 31 (2005) 481
  - Ebert, *et al.*, *PRD* 79 (2009) 114029
- Lattice QCD
  - Dudek, *et al.*, *PRD* 82 (2010) 034508
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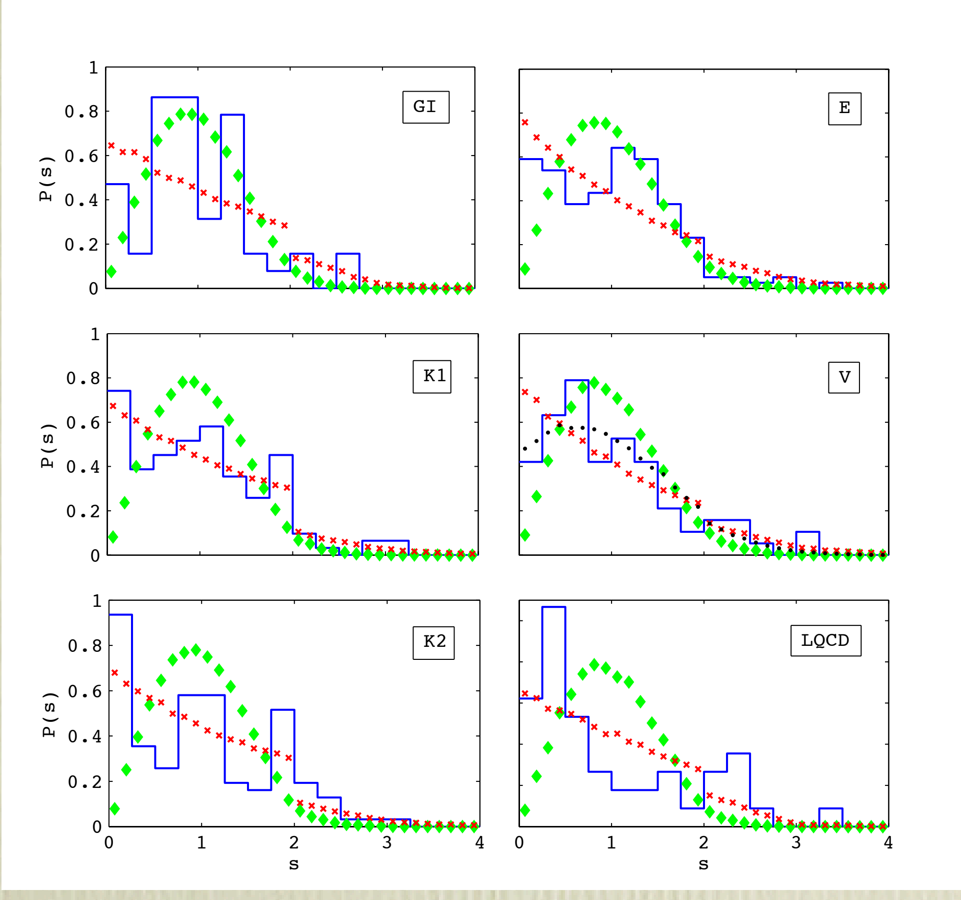

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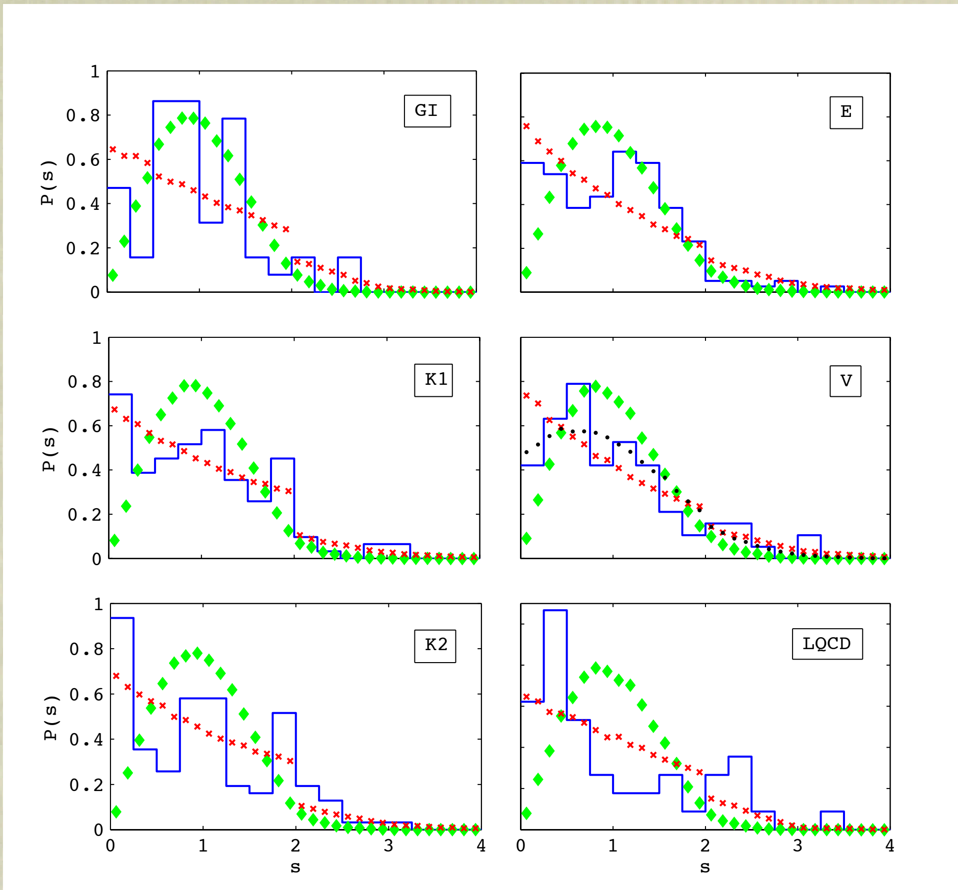

	data	GI	K1	K2	E	V	LQCD
p_{DW}	0.47	0.84	0.038	0.005	0.083	0.51	0.033
p_{DP}	0.13	0.41	0.55	0.43	0.21	0.56	0.44

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    - Gómez, *et al.*, *Phys. Rep.* 499 (2011) 103

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- One can also conclude the importance of correlations in the underlying physics

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- This is consistent with previous analysis of quark models of baryons
  - Fernández-Ramírez, Relaño, *PRL* 98 (2007) 062001

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- Further work is necessary to improve the theoretical models.