Precise determination of the $f_0(600)$ and $f_0(980)$ parameters by fitting the data and dispersion relations

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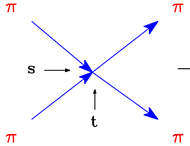
Schedule

- **Theory** - dispersion relations
- **Experiment** - old and present state of art
- **Results** - combined analysis of dispersion relations and data
- **Conclusions**

Dispersion relations for $\pi\pi$ interactions theory $\longleftrightarrow$ experiment

Once subtracted dispersion relations "GKPY" (for the S and P waves):

crossing symmetry:



$\rightarrow \vec{T}_s(s,t) = \hat{C}_{st}\vec{T}_t(t,s)$

$\vec{T}(s,t)$ + crossing symmetry $\rightarrow$ dispersion relations for $4m_\pi^2 < s < \sim (1150\text{ MeV})^2$

$$\text{Re } t_\ell^{I(OUT)}(s) = \sum_{I'=0}^{2} C_{st}^{II'} a_0^{I'} + \sum_{I'=0}^{2}\sum_{\ell'=0}^{4} \fint_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s,s')\,\text{Im } t_{\ell'}^{I'(IN)}(s')$$

$a_0^{I'}$ - subtraction constant $= \vec{T}_s(s = 4m_\pi^2, t = 0)$ - scattering lengths in S wave

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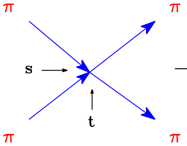

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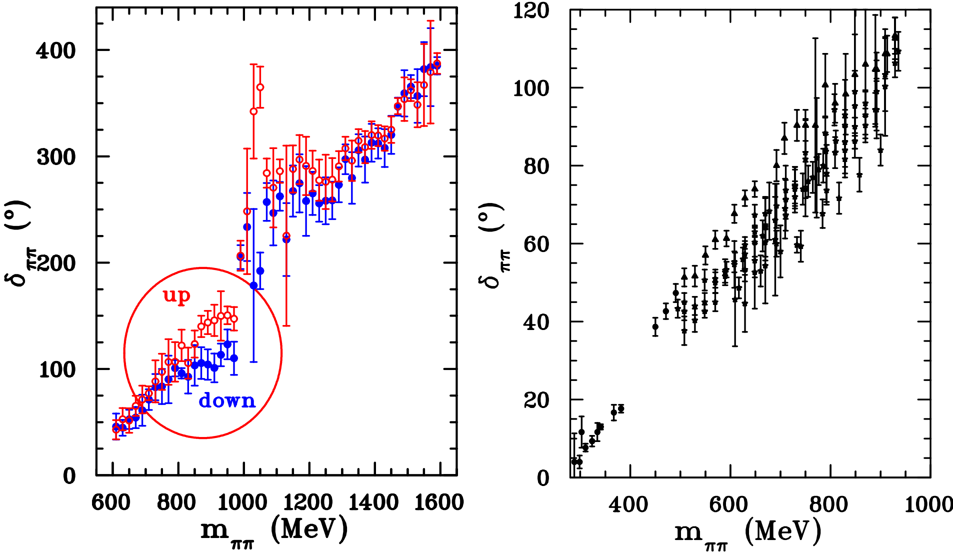
Condition for crossing symmetry: $\text{Re}\; f_\ell^{I(OUT)}(s) - \text{Re}\; f_\ell^{I(IN)}(s) = 0$

Experimental data on the $\pi\pi$ interactions in the ***S0*** wave (notation ***JI***)



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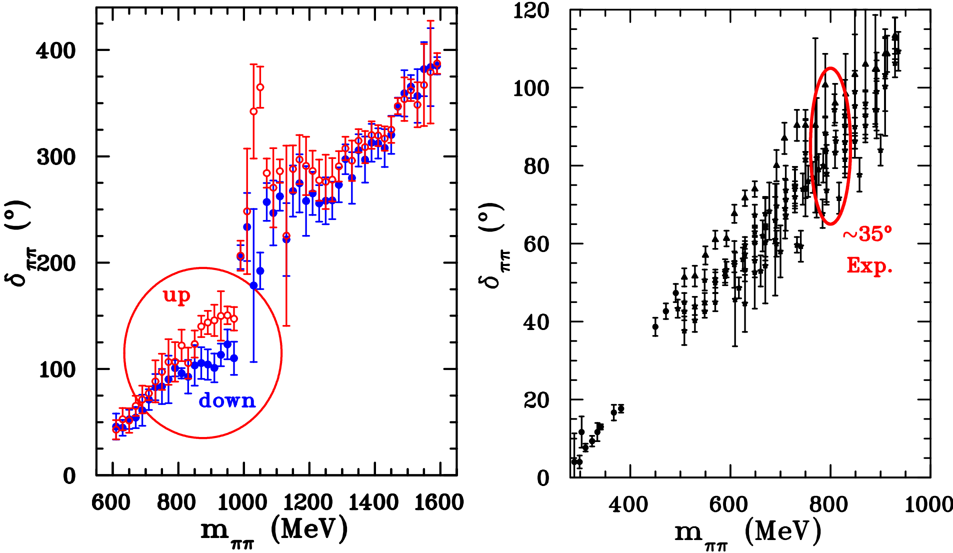

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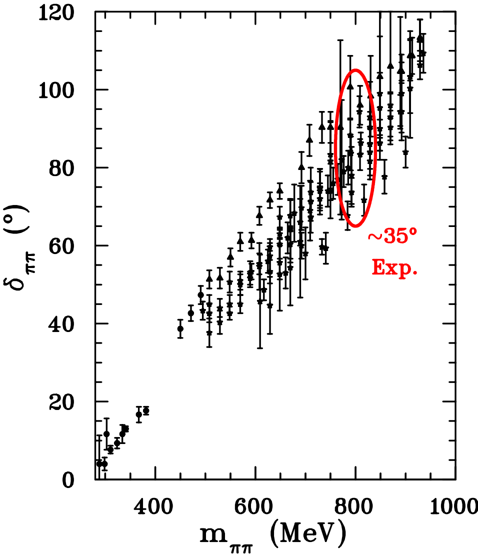
- "up-down" ambiguity eliminated (in 2003) using the Roy equations (Roy 1971) (two subtractions)
- once subtracted dispersion relations GKPY (presented in 2011) - much smaller errors of the output amplitude
- elimination of several sets of experimental data



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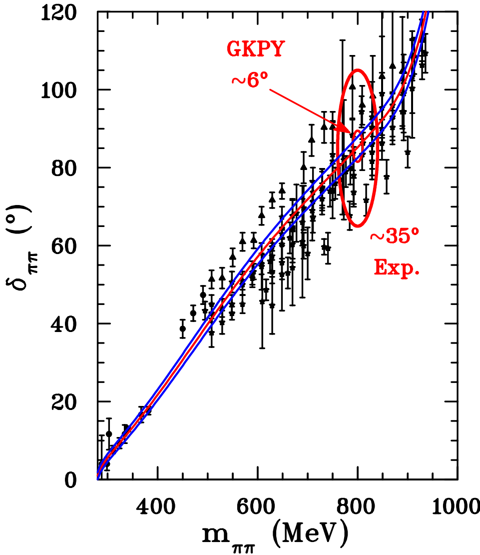
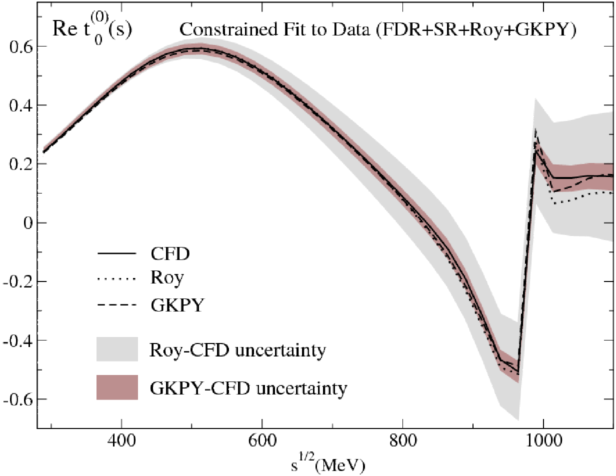


precision of the Roy and GKPY equations



"Precise determination of the f0(600) and f0(980) pole parameters from a dispersive data analysis",
R. Garcia-Martin, R. Kamiński, J.R. Pelaez, J. Ruiz de Elvira,
Phys. Rev. Lett. 107 (2011) 072001

Method of combined analysis (data dispersion relations)

- input amplitudes for the S, P, D and F waves constructed, at the beginning, only by fit to the data,
- just simple polynomials in energy^2,
- no assumption on the low threshold parameters (we use NA48/2 data),
- set of dispersion relations used in the analysis:
  - *once subtracted dispersion relations (GKPY)*,
  - *twice subtracted dispersion relations (Roy)*,
  - *Forward Dispersion Relations (FDR)*,
  - *Olsson sum rule (SR)*,
- phenomenological input partial amplitudes used up to 1420 MeV,
- above 1420 MeV - Regge parameterizations

Method of combined analysis (data dispersion relations)

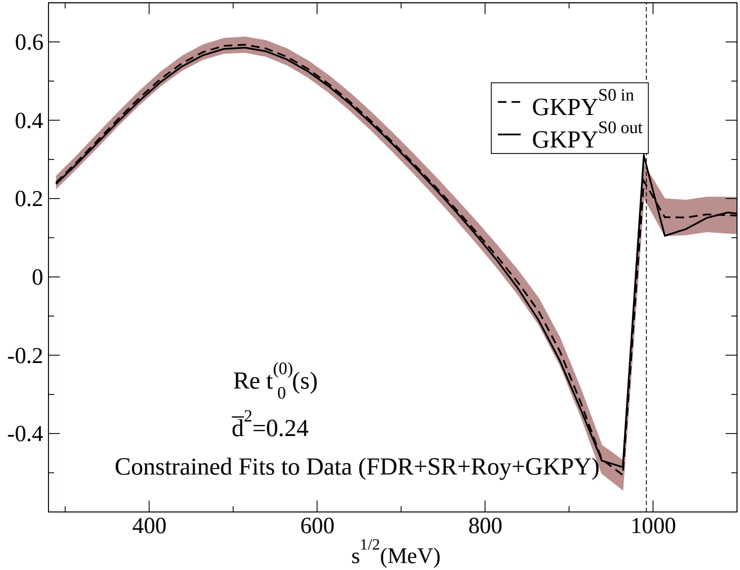
$$\chi^2_{tot} = \chi^2_{data} + \bar{d}^2_{Roy} + \bar{d}^2_{GKPY} + \bar{d}^2_{FDR} + \bar{d}^2_{SR}$$

where $\bar{d}^2_i = \frac{1}{number\ \ of\ \ points} \sum_j^n \left(\frac{\Delta_i(s_j)}{\delta\Delta_i(s_j)} \right)^2$

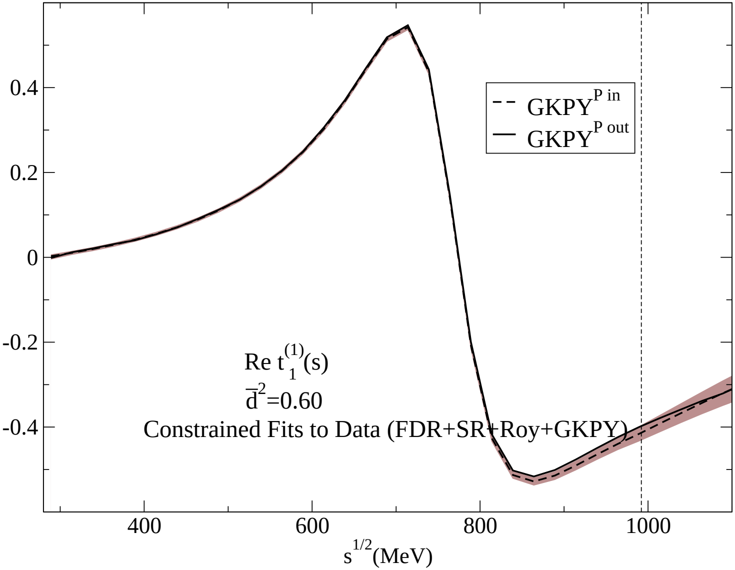
$(number\ \ of\ \ points = 28)$

	fit only to data	fit to data and to dispersion relations
$\bar{d}^2_{Roy}$	0.87	0.14
$\bar{d}^2_{GKPY}$	1.9	0.32
$\bar{d}^2_{FDR}$	1.98	0.4

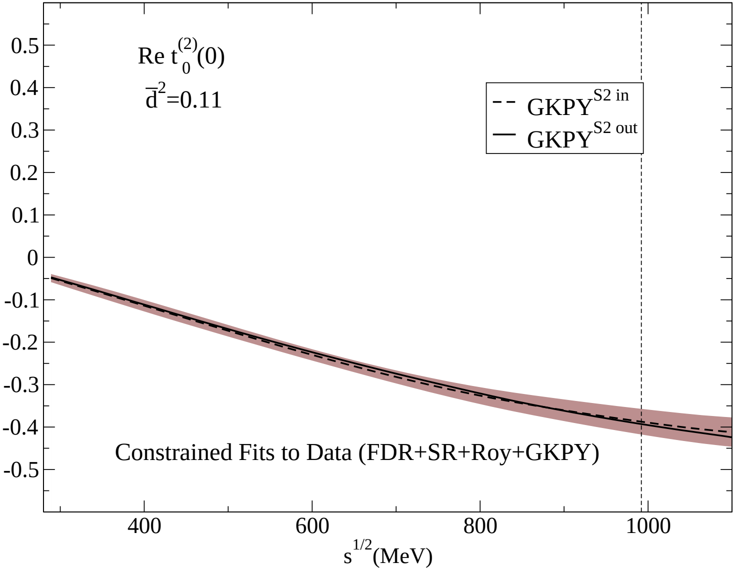
fit to GKPY (S0 wave)



fit to GKPY ($P1$ wave)



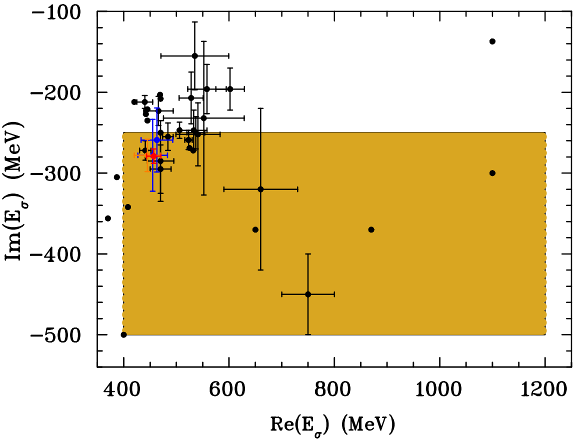
fit to GKPY (S2 wave)



precise determination of $f_0(600)$ (σ) meson and threshold parameters

$f_0(600)$ (σ)

- PDG 2010:
  M = 400 – 1200 MeV
  $\Gamma = 2 \times (250 - 500)$ MeV
- GKPY:
  $E_\sigma = 457 \pm 14 - i279^{+11}_{-7}$ MeV



threshold parameters, e.g. a_0^0:

- ChPT + Roy eqs (Bern group):
  $0.220 \pm 0.005\, m_\pi^{-1}$
- GKPY:
  $0.220 \pm 0.008\, m_\pi^{-1}$

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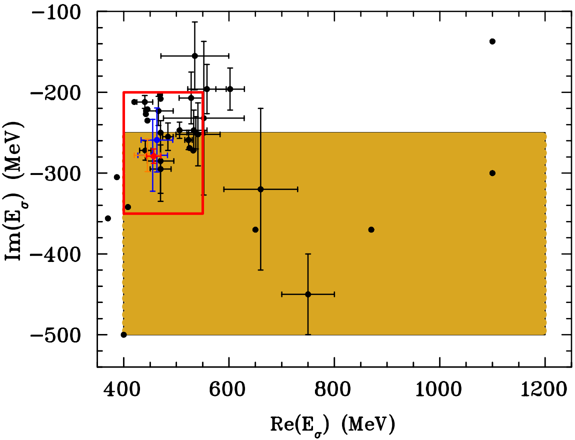

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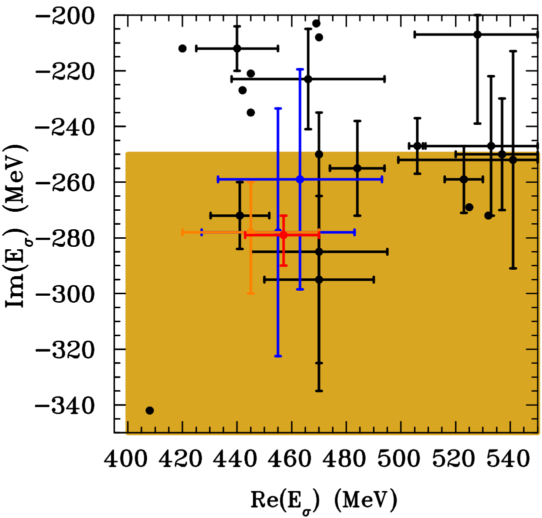
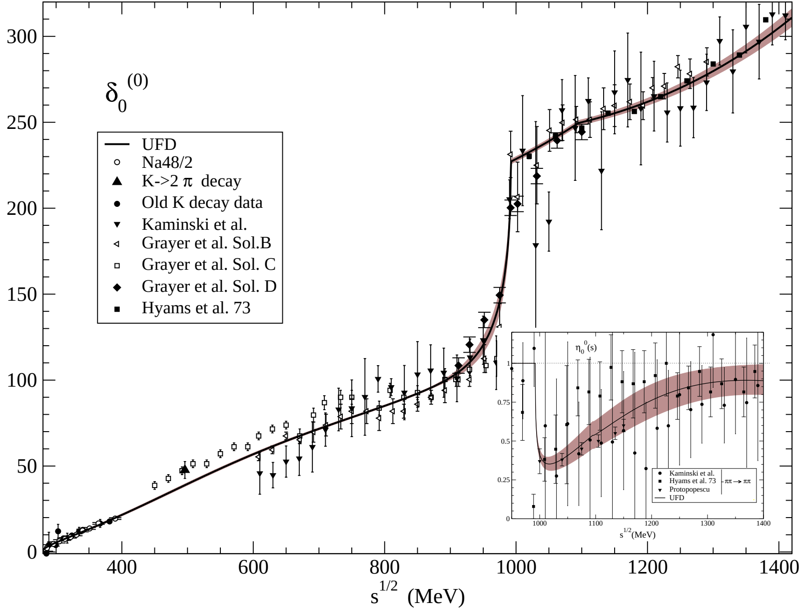


precise determination of $f_0(980)$ meson



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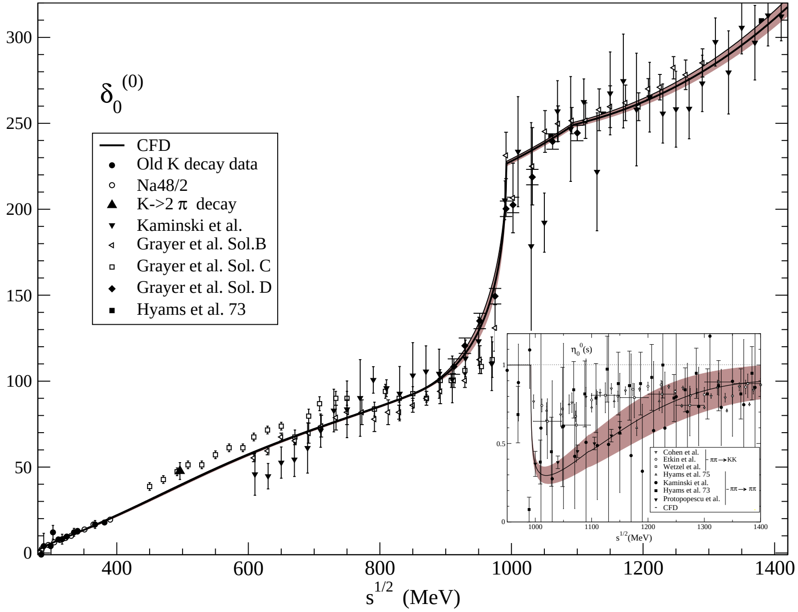

precise determination of $f_0(980)$ meson

Model-independent analytic continuation to the complex plane gets pole at $s_{pole}^{1/2}$ on the 2nd Riemann sheet

assuming that $M = Re(s_{pole}^{1/2})$ and $\Gamma = -2\,Im(s_{pole}^{1/2})$ we get:

$M_{f_0(980)} = 996 \pm 7$ MeV and

$\Gamma_{f_0(980)} = 50^{+20}_{-12}$ MeV

PDG'2010: Mass $m = 980 \pm 10$ MeV

Width $\Gamma = 40 - 100$ MeV

precise determination of couplings to the $\pi\pi$ channel

$$g^2 = -16\pi \lim_{s \to s_{pole}} (s - s_{pole})\, t_\ell(s)\, (2\ell + 1)/(2p)^{2\ell}$$

where $p^2 = s/4 - m_\pi^2$.

	$\sqrt{s_{\rm pole}}$ (MeV)	$\|g\|$
$f_0(600)^{\rm GKPY}$	$(457^{+14}_{-13}) - i(279^{+11}_{-7})$	$3.59^{+0.11}_{-0.13}$ GeV
$f_0(600)^{\rm Roy}$	$(445 \pm 25) - i(278^{+22}_{-18})$	3.4 ± 0.5 GeV
$f_0(980)^{\rm GKPY}$	$(996 \pm 7) - i(25^{+10}_{-6})$	2.3 ± 0.2 GeV
$f_0(980)^{\rm Roy}$	$(1003^{+5}_{-27}) - i(21^{+10}_{-8})$	$2.5^{+0.2}_{-0.6}$ GeV
$\rho(770)^{\rm GKPY}$	$(763.7^{+1.7}_{-1.5}) - i(73.2^{+1.0}_{-1.1})$	$6.01^{+0.04}_{-0.07}$
$\rho(770)^{\rm Roy}$	$(761^{+4}_{-3}) - i(71.7^{+1.9}_{-2.3})$	$5.95^{+0.12}_{-0.08}$

Conclusions

- due to works on once and twice subtracted dispersion relations with imposed crossing symmetry condition we have in disposal very efficient set of rules for testing the partial $\pi\pi$ amplitudes in the S, P, D and F waves,
- we also have set of model independent unitary $\pi\pi$ amplitudes in those waves in the range from $2m_\pi$ to several GeV fulfilling very well crossing symmetry below $\sim$ 1100 MeV,
- as an artefact we got very precise values of parameters for the $f_0(600)$ and $f_0(980)$ resonances