A Dispersive Treatment of $K_{\ell 4}$ Decays

Peter Stoffer

stoffer@itp.unibe.ch

Work in collaboration with G. Colangelo and E. Passemar

Albert Einstein Center for Fundamental Physics
Institute for Theoretical Physics
University of Bern

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Outline

1. Motivation
2. Dispersion Relation for $K_{\ell 4}$ Decays
3. Results
4. Outlook

Importance of $K_{\ell 4}$ decays

Unique information about some low energy constants of ChPT:

- L_1^r, L_2^r, L_3^r multiply operators with four derivatives $\Rightarrow$ We need a four-“particle” process
- $K_{\ell 4}$ like a $2 \to 2$ scattering
- Happens at low energy, where ChPT is expected to converge better

Importance of $K_{\ell 4}$ decays

- Provides information on $\pi\pi$ scattering lengths a_0^0, a_0^2
- Very precisely measured $\Rightarrow$ Test of ChPT

  $\rightarrow$ Geneva-Saclay, E865, NA48/2
- Kaon physics: High precision at low energy as a key to new physics?

  $\rightarrow$ NA62

Advantages of dispersion relations

- Summation of rescattering
- Connects different energy regions
- Based on analyticity and unitarity $\Rightarrow$ Model independence
- $\mathcal{O}(p^6)$ result available, but only useful if LECs are known

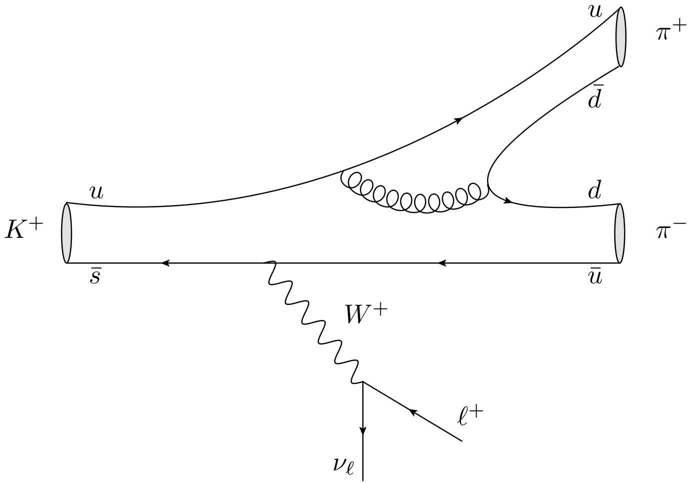
$K_{\ell 4}$ decays

Decay of a kaon in two pions and a lepton pair:

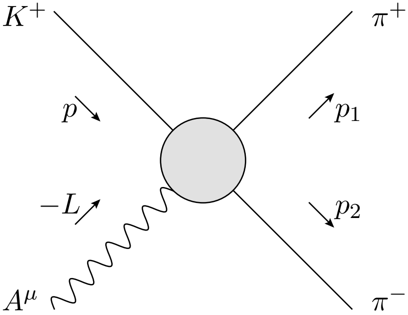
$$K^+(p) \to \pi^+(p_1)\pi^-(p_2)\ell^+(p_\ell)\nu_\ell(p_\nu)$$

$\ell \in \{e, \mu\}$ is either an electron or a muon.

SM tree-level



Hadronic part of $K_{\ell 4}$ as $2 \to 2$ scattering



Form factors

- Lorentz structure allows four form factors in the hadronic matrix element.

$$\left\langle \pi^+(p_1)\pi^-(p_2) \middle| V_\mu(0) \middle| K^+(p) \right\rangle = -\frac{H}{M_K^3}\epsilon_{\mu\nu\rho\sigma} L^\nu P^\rho Q^\sigma$$

$$\left\langle \pi^+(p_1)\pi^-(p_2) \middle| A_\mu(0) \middle| K^+(p) \right\rangle = -i\frac{1}{M_K}\left(P_\mu F + Q_\mu G + L_\mu R\right)$$

- In experiments, just K_{e4} decays are measured, yet. There, mainly one specific linear combination $F_1(s,t,u)$ of the form factors F and G is accessible.

Analytic properties

- $F_1(s,t,u)$ has a right-hand branch cut in the complex s-plane, starting at the $\pi\pi$-threshold.
- Left-hand cut present due to crossing.
- Analogous situation in t- and u-channel.

Decomposition into functions of a single variable

Decomposition has been done first for the $\pi\pi$ scattering amplitude.

$\rightarrow$ Stern, Sazdjian, Fuchs (1993)

Define a function that has just the right-hand cut of the partial wave f_0:

$$M_0(s) := P(s) + \frac{s^4}{\pi} \int_{4M_\pi^2}^{\Lambda^2} \frac{\mathrm{Im} f_0(s')}{(s' - s - i\epsilon){s'}^4} ds'$$

Decomposition into functions of a single variable

Define similar functions that take care of the right-hand cuts of f_1 and the S- and P-waves in the crossed channels.

All the discontinuities are split up into functions of a single variable. $\Rightarrow$ Major simplification!

Decomposition into functions of a single variable

We neglect:

- Imaginary parts of D- and higher waves,
- High energy tail of dispersion integral from Λ^2 to ∞.

Both effects are of $\mathcal{O}(p^8)$.

Decomposition into functions of a single variable

Respecting isospin properties, we end up with the following decomposition:

$$\begin{aligned} F_1(s,t,u) &= M_0(s) + \frac{2}{3}N_0(t) + \frac{1}{3}R_0(t) + R_0(u) \\ &+ (u-t)M_1(s) - \frac{2}{3}\Big[t(u-s) - \Delta_{K\pi}\Delta_{\ell\pi}\Big]N_1(t) \\ &+ \mathcal{O}(p^8). \end{aligned}$$

Dispersion relation

Solution of the Omnès problem:

$$M_0(s) = \Omega_0^0(s)\left\{P(s) + \frac{s^3}{\pi}\int_{4M_\pi^2}^{\Lambda^2} \frac{\hat{M}_0(s')\sin\delta_0^0(s')}{|\Omega_0^0(s')|(s'-s-i\epsilon){s'}^3}ds'\right\},$$

with the Omnès function

$$\Omega_0^0(s) := \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\delta_0^0(s')}{s'(s'-s-i\epsilon)}\,ds'\right\}.$$

Similar relations for the other functions.

Phase inputs

We need the following phase shifts:

- δ_0^0, δ_1^1: $\pi\pi$ scattering
- $\delta_0^{1/2}$, $\delta_1^{1/2}$, $\delta_0^{3/2}$: $K\pi$ scattering

(δ_l^I: l – angular momentum, I – isospin)

Hat functions

- The left-hand cut is contained in $\hat{M}_0(s)$.
- $\hat{M}_0(s)$ is given as angular averages of N_0, N_1, $\ldots$

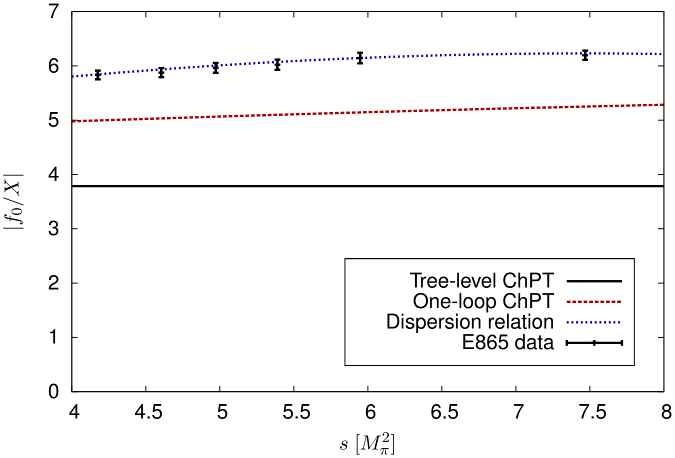
Intermediate summary

- Problem parametrised by five subtraction constants.
- Elastic scattering phase shifts as inputs.
- Energy dependence fully determined by the dispersion relation.

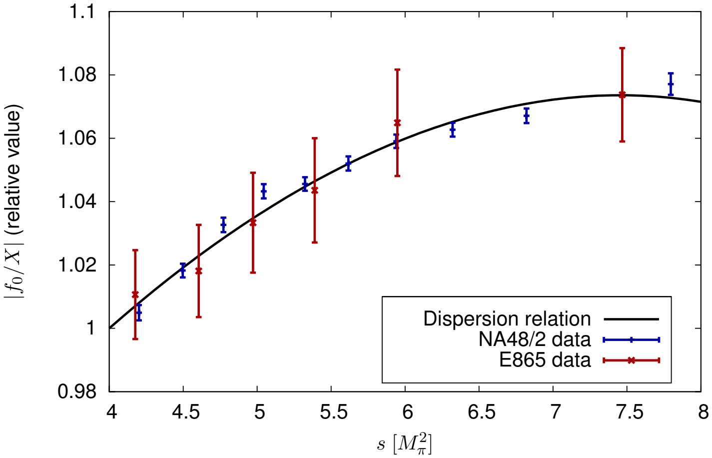
Intermediate summary

- Coupled set of integral equations:
  $\Rightarrow M_0(s), M_1(s), \ldots$: DR involving $\hat{M}_0(s), \hat{M}_1(s), \ldots$
  $\Rightarrow \hat{M}_0(s), \hat{M}_1(s), \ldots$: Angular integrals over $M_0(s), M_1(s), \ldots$
- System solved by iteration
- Problem linear in subtraction constants $\Rightarrow$ Fit data with a linear combination of five basic solutions

Fit of the S-wave



Fit of the S-wave



Determination of LECs

- Matching the dispersive result to ChPT at $s = t - u = 0$: Below threshold, where ChPT converges better
- L_1^r, L_2^r and L_3^r can be determined

Work in progress

- Isospin corrections
- Matching to $\mathcal{O}(p^6)$ ChPT

Summary

- Parametrisation valid up to and including $\mathcal{O}(p^6)$
- Model independence
- Full summation of rescattering effects
- Very precise data available
- Advantage over pure ChPT: Matching below threshold, where ChPT converges better $\Rightarrow$ LECs

Determination of LECs - preliminary!

Fit to NA48/2 (partial sample) with E865 norm, matching to $\mathcal{O}(p^4)$ ChPT ($\mu = 770$ MeV):

$$L_1^r = (0.72 \pm 0.29) \cdot 10^{-3}$$
$$L_2^r = (0.64 \pm 0.27) \cdot 10^{-3}$$
$$L_3^r = (-2.71 \pm 1.18) \cdot 10^{-3}$$

J. Bijnens, I. Jemos, 'fit All': $\rightarrow$ arXiv:1103.5945 [hep-ph]

$$L_1^r = (0.88 \pm 0.09) \cdot 10^{-3}$$
$$L_2^r = (0.61 \pm 0.20) \cdot 10^{-3}$$
$$L_3^r = (-3.04 \pm 0.43) \cdot 10^{-3}$$