Radiative decays of vector and pseudoscalar nonets

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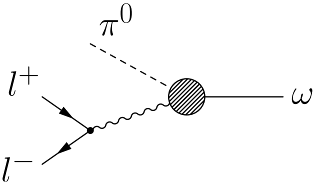
Collaborators: S. Leupold, M.F.M. Lutz

Talk based on arXiv:1204.4125 [hep-ph].

Motivation

Consider the decay $\omega \to \pi^0 l^+ l^-$.



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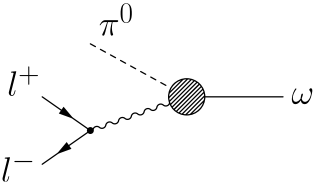
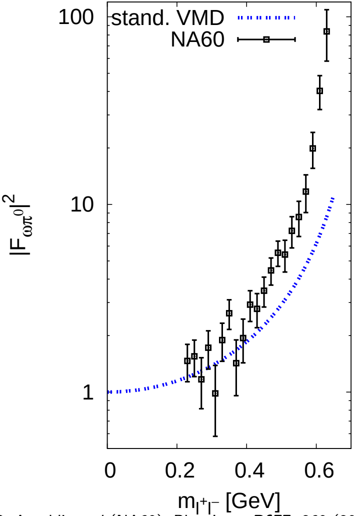
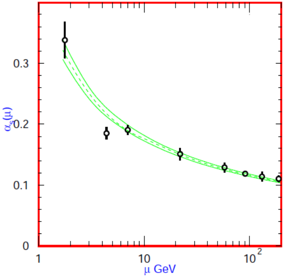




R. Arnaldi *et al* (NA60), Phys.Lett. **B677**, 260 (2009)

$\hookrightarrow$ standard vector meson dominance (VMD) fails to describe the data

Problem in QCD



PDG, J. Phys. **G33**, 1 (2006)

Running coupling constant in QCD

- high energies:
  can use perturbation theory
- low energies:
  cannot use perturbation theory

Possible solution:

effective theories

↪ hadrons as relevant degrees of freedom

Effective theories for light mesons

Chiral perturbation theory (ChPT): vector mesons are heavy

$\Rightarrow$ not applicable for energy range of hadronic resonances (ρ, ω, K^*, φ, η')

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- masses of both vector mesons and pseudoscalar mesons are treated as soft, i.e. $\sim Q$
- decays: all involved momenta are smaller than the mass of the decaying meson, i.e. $\sim Q$

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Possible justification:

other low-lying mesons are dynamically generated from interactions of pseudoscalar and vector mesons (hadrogenesis conjecture)

Leading-order Lagrangian

Leading-order (LO) Lagrangian for decays $V \to P\gamma^*$ and $P \to V\gamma^*$:

$$\begin{aligned}\mathcal{L} = &- \frac{h_A}{16f}\,\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\left\{\left[\Phi_{\mu\nu}, \partial^\tau \Phi_{\tau\alpha}\right]_+ \partial_\beta \Phi\right\} \\ &- \frac{b_A}{8f}\,\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\left\{\Phi_{\mu\nu}\left[\Phi, \chi_0\right]_+ \Phi_{\alpha\beta}\right\} \\ &- \frac{m_V^2 h_H}{4 f_H}\,\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\left\{\Phi_{\mu\nu}\Phi_{\alpha\beta}\right\}\tilde{\eta}_1 \\ &- e f_V\,\mathrm{tr}\left\{\Phi^{\mu\nu}\mathcal{Q}\right\}\partial_\mu A_\nu \\ &+ \frac{e f_V e_H}{f_H}\,\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\left\{\Phi_{\mu\nu}\,\mathcal{Q}\right\}\partial_\alpha A_\beta\,\tilde{\eta}_1\end{aligned}$$

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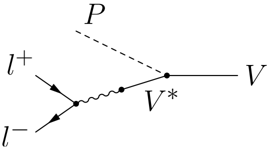

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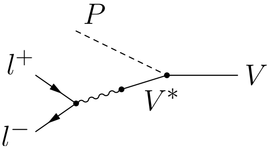

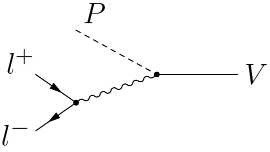

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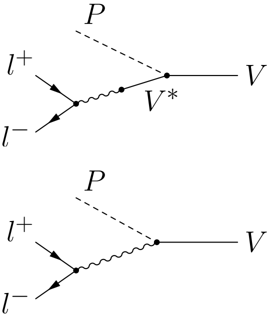

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Decay photon into dilepton: usual QED

η - η' mixing

Pseudoscalar meson nonet Φ includes non-physical singlet state $\tilde{\eta}_1$ and octet state η_8:

$$\Phi = \eta_8\,\lambda^8 + \tilde{\eta}_1\,\frac{f}{f_H}\,\lambda^0 + \dots$$

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$\Rightarrow$ use θ as additional open parameter

Parameter determination (I)

LO Lagrangian has **six open parameters** h_A, b_A, h_H, e_H, f_H and θ:

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$\hookrightarrow$ determine all parameters except f_H from **five two-body decays**

$$\omega \to \pi^0\gamma, \quad \omega \to \eta\gamma, \quad \phi \to \eta\gamma, \quad \phi \to \eta'\gamma, \quad \eta' \to \omega\gamma$$

$\Rightarrow$ compare $\left|\mathcal{M}_{A\to B\gamma}\right|^2$ with $\left|\mathcal{M}^{\mathrm{exp}}_{A\to B\gamma}\right|^2$

Parameter determination (II)

only absolute values $|\mathcal{M}_{A\to B\gamma}|$ can be compared

$\Rightarrow$ after fixing h_A to be positive: four parameter sets left

$\Rightarrow$ **two sets with "reasonable" parameters** (compared to previous calculations without η')

M. F. M. Lutz, S. Leupold, Nucl. Phys. **A813**, 96 (2008).
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$$\begin{aligned} \theta &= \pm 2.0^\circ, \\ h_A &= 2.33, \\ b_A &= 0.16, \\ h_H &= 0.14 \mp 0.19\,\frac{f_H}{f}, \\ e_H &= -0.20 \mp 0.70\,\frac{f_H}{f} \end{aligned}$$

with $f_H \in [f, \sqrt{2} f]$

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For decays into dileptons: no additional parameters needed

$\Rightarrow$ predictive power

Decay $\omega \to \pi^0 \, l^+ l^-$ (I)

Isospin conservation: decay only possible via virtual ρ^0-meson

$\Rightarrow$ **standard VMD** form factor (with invariant dilepton mass q):

$$F_{\omega\pi^0}^{\mathrm{VMD}}(q) = \frac{m_\rho^2}{m_\rho^2 - q^2}$$

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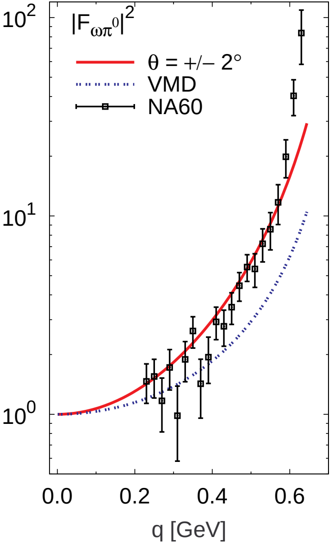
$\hookrightarrow$ **our calculations** yield an additional constant term:

$$F_{\omega\pi^0}(q) \sim h_A + \frac{-\left(m_\omega^2 + m_\rho^2\right) h_A + 8\, b_A\, m_\pi^2}{m_\rho^2 - q^2}$$

Decay $\omega \rightarrow \pi^0 l^+ l^-$ (I)



(Data taken by NA60 for $\omega \rightarrow \pi^0 \mu^+ \mu^-$)

- standard VMD fails to explain data
- our calculations miss only the last three data points
- reduced χ^2 for single-differential decay width:
  $\chi^2_{\text{our theo.}} = 1.8$ and $\chi^2_{\text{VMD}} = 4.8$

$$\Gamma_{\omega\rightarrow\pi^0\mu^+\mu^-} = (9.74 \pm 0.30) \cdot 10^{-7}\,\text{GeV}$$
$$\Gamma^{\text{exp}}_{\omega\rightarrow\pi^0\mu^+\mu^-} = (11.04 \pm 3.40) \cdot 10^{-7}\,\text{GeV}$$

$$\Gamma_{\omega\rightarrow\pi^0 e^+e^-} = (6.85 \pm 0.21) \cdot 10^{-6}\,\text{GeV}$$
$$\Gamma^{\text{exp}}_{\omega\rightarrow\pi^0 e^+e^-} = (6.54 \pm 0.51) \cdot 10^{-6}\,\text{GeV}$$

Decay $\eta' \to \omega e^+ e^-$

Take into account η - η' mixing

$\hookrightarrow$ Form factor:

$$f_{\omega\eta'} = \sin\theta \, f_{\eta_8\omega} + \frac{f}{f_H} \cos\theta \, f_{\eta_1\omega}$$

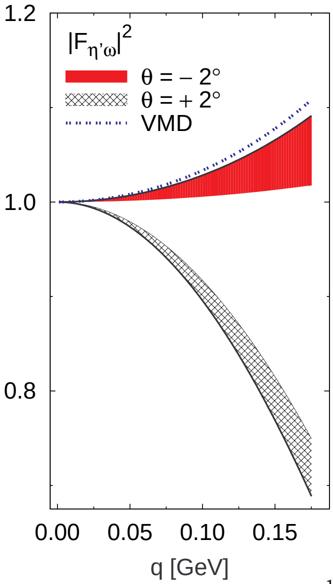
Decay $\eta' \to \omega e^+ e^-$

Take into account η - η' mixing

$\hookrightarrow$ Form factor:

$$f_{\omega\eta'} = \sin\theta f_{\eta_8\omega} + \frac{f}{f_H}\cos\theta f_{\eta_1\omega}$$

$\hookrightarrow$
- clear deviation for $\theta = \pm 2°$
- uncertainty caused by f_H
- (clear) deviation from VMD



Prediction:

$$\text{Br}_{\eta' \to \omega e^+ e^-} = (1.69 \pm 0.56) \cdot 10^{-4}$$

Summary and Outlook

- introduced new counting scheme which treats nonets of pseudoscalar and vector mesons on same footing
- partial decay widths $\Gamma_{A \to B l^+ l^-}$ are in good agreement with the available experimental data
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Summary and Outlook

- introduced new counting scheme which treats nonets of pseudoscalar and vector mesons on same footing
- partial decay widths $\Gamma_{A\to Bl^+l^-}$ are in good agreement with the available experimental data
- $\omega \to \pi^0$ transition form factor much better described than with VMD
- additional decays calculated as, e.g., $\pi^0/\eta/\eta' \to \gamma l^+ l^- / 2l^+ 2l^-$ (with $e_H, h_H = 0$ so far) and $\omega \to 3\pi$
- **next step:** next-to-leading order calculations

Thanks for your attention.

Additional slides.

The counting scheme

Problem: infinite number of interactions terms and parameters in the chiral Lagrangian

$\hookrightarrow$ relevance of each term needed to make predictions

$\Rightarrow$ counting scheme

To determine counting rules:

identify soft scale Λ_{soft} and hard scale Λ_{hard} (separation of scales)

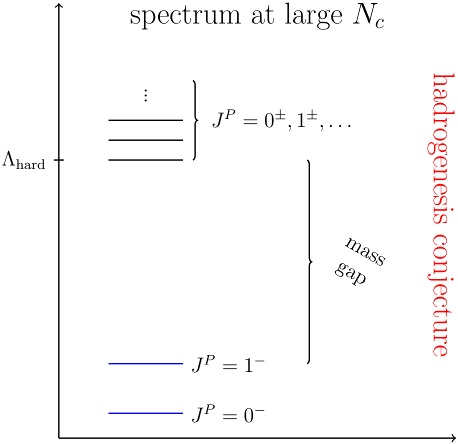
$\hookrightarrow$ can expand Lagrangian in terms of $\Lambda_{\text{soft}}/\Lambda_{\text{hard}}$

In general: soft scale $=$ masses of particles taken as relevant degrees of freedom (DOF)

hard scale $=$ masses of particles not involved as DOF

Difficulty: identification of scales, especially if particles are generated dynamically by DOF

Idea: take light pseudoscalar and vector mesons as DOF
$\hookrightarrow$ soft scale $=$ masses of pseudoscalars and vectors



Hadrogenesis conjecture:
mass gap between DOF and other mesons

In the large-N_c limit:
couplings are zero
$\Rightarrow$ no dynamically generated particles

For $N_C = 3$:

other low-lying mesons are dynamically generated by interactions of pseudoscalar and vector mesons

$\hookrightarrow$ leading-order interaction generates e.g. axial-vector resonances quite well

M. F. M. Lutz, E. E. Kolomeitsev, Nucl. Phys. **A730**, 392 (2004)

$\Rightarrow$ if one relies on hadrogenesis and takes light pseudoscalars and vector mesons as DOF, the counting rules are given by

$$\boxed{D_\mu,\, m_P,\, m_V \sim Q.}$$

$\hookrightarrow$ relies on a sufficiently large hard scale $\Lambda_{\text{hard}} \geq (2-3)$ GeV

Additionally:
- suppression of n-body forces $\sim N_c^{1-n/2}$
- OZI suppression of additional traces (except the pseudoscalar singlet)

$\hookrightarrow$ keep large-N_c behaviour which can be seen at $N_c = 3$

Representation of vector and pseudoscalar fields

Use counting scheme to derive leading-order Lagrangian for pseudoscalar mesons

$$\Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 \end{pmatrix} + \tilde{\eta}_1 \frac{f}{f_H}\sqrt{\frac{2}{3}} I_{3\times 3}$$

and light vector mesons (described by antisymmetric tensor fields)

$$\Phi_{\mu\nu} = \begin{pmatrix} \rho^0_{\mu\nu} + \omega_{\mu\nu} & \sqrt{2}\rho^+_{\mu\nu} & \sqrt{2}K^+_{\mu\nu} \\ \sqrt{2}\rho^-_{\mu\nu} & -\rho^0_{\mu\nu} + \omega_{\mu\nu} & \sqrt{2}K^0_{\mu\nu} \\ \sqrt{2}K^-_{\mu\nu} & \sqrt{2}\bar{K}^0_{\mu\nu} & \sqrt{2}\varphi_{\mu\nu} \end{pmatrix}$$

η - η' mixing (detailed) (I)

Pseudoscalar meson nonet Φ includes non-physical singlet state $\tilde{\eta}_1$ and octet state η_8:

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$$= \eta_8 \lambda^8 + \tilde{\eta}_1 \frac{f}{f_H} \lambda^0$$

$\Rightarrow$ physical fields η and η' are defined via

$$\begin{aligned} \eta &= -\tilde{\eta}_1 \sin\theta + \eta_8 \cos\theta, \\ \eta' &= \tilde{\eta}_1 \cos\theta + \eta_8 \sin\theta \end{aligned}$$

with to be determined mixing angle θ

η - η' mixing (detailed) (II)

LO kinetic and mass terms for the pseudoscalar mesons:

$$\mathcal{L}_{\text{pseudo}} = -\frac{1}{4}\,\text{tr}\,\{\partial_\mu \Phi\, \partial^\mu \Phi\} + \frac{1}{2}\left(1 - \frac{f^2}{f_H^2}\right) \partial_\mu \tilde{\eta}_1\, \partial^\mu \tilde{\eta}_1 - \frac{1}{2}\, m_H^2 \tilde{\eta}_1{}^2$$
$$- \frac{1}{4}\,\text{tr}\,\{\Phi \chi_0 \Phi\} + \frac{f\, b_H}{\sqrt{6} f_H}\,\text{tr}\,\{\Phi \chi_0\}\, \tilde{\eta}_1 + \frac{f^2 g_0}{2 f_H^2}\,\text{tr}\,\{\chi_0\}\, \tilde{\eta}_1{}^2$$

Definition: no mixing between η and η'

$\Rightarrow$ no mass terms proportional to $\eta \cdot \eta'$ in the Lagrangian $\mathcal{L}_{\text{pseudo}}$

$\Rightarrow$ relation for mixing angle θ:

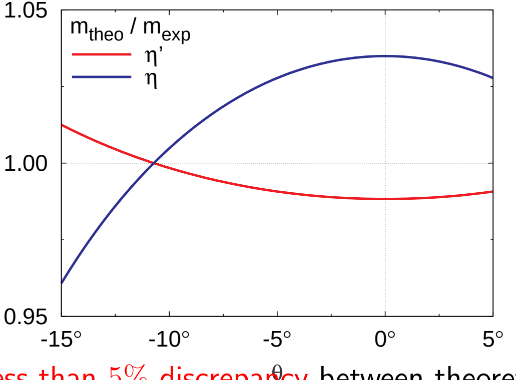
$$\cos(2\theta) = \frac{m_{\eta'}^2 + m_\eta^2 - \frac{2}{3}\left(4 m_K^2 - m_\pi^2\right)}{m_{\eta'}^2 - m_\eta^2}$$

$\Rightarrow$ $\theta = -10.7^\circ$, independent of f_H

"Literature" value $\theta = -20^\circ$ for $b_H = 0$ and $f_H = f$.

η - η' mixing (detailed) (III)

Alternative: use relation for the mixing angle to calculate η'-mass and η-mass as functions of θ and $\left(m_{\eta'}^2 + m_\eta^2\right)^{\text{exp}}$.



⇒ less than 5% discrepancy between theoretical and experimental masses for $|\theta| \leq 15°$

⇒ use θ as additional open parameter

Why we don't fix θ

Mixing angle θ could be fixed by mass term for pseudoscalar mesons
$\Rightarrow$ can determine **"experimental width"** for decays into η_1 and η_8:

$$\Gamma_{\eta_8} = \cos^2\theta\,\Gamma_\eta + \sin^2\theta\,\Gamma_{\eta'} + 2\sin\theta\,\cos\theta\,\sqrt{\Gamma_\eta\,\Gamma_{\eta'}},$$
$$\frac{f_H^2}{f^2}\,\Gamma_{\eta_1} = \sin^2\theta\,\Gamma_\eta + \cos^2\theta\,\Gamma_{\eta'} - 2\sin\theta\,\cos\theta\,\sqrt{\Gamma_\eta\,\Gamma_{\eta'}}$$

$\hookrightarrow$ Γ_{η_8} depends only on h_A and b_A, Γ_{η_1} depends on all parameters

$\hookrightarrow$
- h_A and b_A determined by $\omega \to \pi^0\gamma$, $\omega \to \eta_8\gamma$ and $\phi \to \eta_8\gamma$
  $\Rightarrow$ overdetermined, no good description of all three decays
- e_H, h_H and f_H determined by $\omega \to \eta_1\gamma$ and $\phi \to \eta_1\gamma$
  $\Rightarrow$ underdetermined, i.e. unfixed parameter

Decay $\eta' \to \omega e^+ e^-$ (details)

- only virtual ω meson possible
- additional contributions from terms proportional to h_H and e_H
- take η-η' mixing into account: $f_{\omega\eta'} = \sin\theta\, f_{\eta_8\omega} + \frac{f}{f_H} \cos\theta\, f_{\eta_1\omega}$

Again, one gets an additional constant term (compared to VMD):

$$f_{\eta_8\omega}(q) \sim h_A + \frac{-2m_\omega^2\, h_A + 8m_\pi^2\, b_A}{m_\omega^2 - q^2},$$

$$f_{\eta_1\omega}(q) \sim \sqrt{2}\left\{ h_A - 2\sqrt{6}\, e_H + \frac{-2m_\omega^2\, h_A + 8m_\pi^2\, b_A + 4\sqrt{6}\, m_V^2\, h_H}{m_\omega^2 - q^2} \right\}.$$

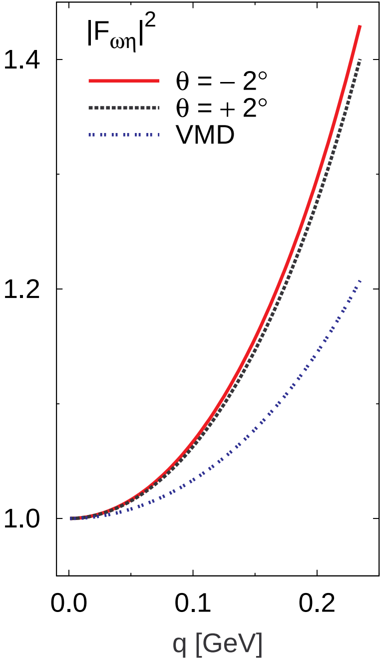
Decay $\omega \to \eta l^+ l^-$

- only virtual ω meson possible
- additional contributions from terms proportional to h_H and e_H
- take η-η' mixing into account: $f_{\omega\eta} = \cos\theta\, f_{\omega\eta_8} - \frac{f}{f_H} \sin\theta\, f_{\omega\eta_1}$

Again, one gets an additional constant term (compared to VMD):

$$f_{\omega\eta_8}(q) \sim h_A + \frac{-2m_\omega^2\, h_A + 8m_\pi^2\, b_A}{m_\omega^2 - q^2},$$

$$f_{\omega\eta_1}(q) \sim \sqrt{2}\left\{h_A - 2\sqrt{6}\, e_H + \frac{-2m_\omega^2\, h_A + 8m_\pi^2\, b_A + 4\sqrt{6}\, m_V^2\, h_H}{m_\omega^2 - q^2}\right\}.$$



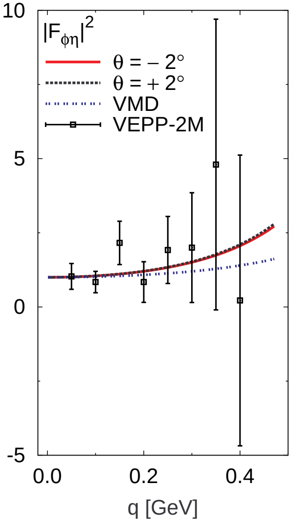
- form factor also depends on h_H, e_H and θ
  $\hookrightarrow$ difference between parameter sets for $\theta = \pm 2°$
- deviation from VMD

Predictions:

Br$_{\omega\to\eta\mu^+\mu^-} = (1.01 \pm 0.08) \cdot 10^{-9}$

Br$_{\omega\to\eta e^+e^-} = (3.39 \pm 0.26) \cdot 10^{-6}$

Decay $\phi \to \eta\, l^+ l^-$



Data taken at VEPP-2M for $\phi \to \eta e^+ e^-$
M.N. Achasov *et al.*, Phys. Lett **B504**, 275 (2001)

- data show relatively large error bars $\Rightarrow$ no assessment possible
- our theory shows deviation from VMD

$$\Gamma_{\phi\to\eta\mu^+\mu^-} = (2.83 \pm 0.33) \cdot 10^{-8}\,\text{GeV}$$
$$\Gamma^{\text{exp}}_{\phi\to\eta\mu^+\mu^-} < 4.00 \cdot 10^{-8}\,\text{GeV}$$

$$\Gamma_{\phi\to\eta e^+e^-} = (4.81 \pm 0.59) \cdot 10^{-7}\,\text{GeV}$$
$$\Gamma^{\text{exp}}_{\phi\to\eta e^+e^-} = (4.90 \pm 0.43) \cdot 10^{-8}\,\text{GeV}$$